Indian Summer School of Physics: Phenomenology of Hot and Dense Matter for Future Accelerators

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Hadron structure phenomenology at an EIC

Problems:

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1. Coherence and shadowing: the two-scattering case

Consider a high-energy massless scalar projectile scattering on some centers located in positions $x_1, x_2, \dots$ - a nucleus. Use light-cone coordinates $a_\pm = (a_0 \pm a_z)/\sqrt{2}$, $a = (a_0, a_T, a_z) = (a_+, a_-, a_T)$ with $a_T = (a_x, a_y)$ the two-dimensional transverse vector, assume dominance of the +-components for the projectile, and define $q = p' - p$. Employ the optical theorem for purely imaginary amplitudes, $it(q = 0) = it_{\rm forw} = -\sigma/2$ for projectile-nucleon and $i\mathcal{T}_n(q = 0) = -\sigma_A^n/2$ for the n-scattering contribution for projectile-nucleus collisions, and the Feynman rules shown in Fig. 1. Then the amplitude with one scattering (Fig. 1 left) reads:

$$
\begin{aligned}
c(p_+, p'_+) i\mathcal{T}_1(q) &= it_{\rm forw}\ A(p_+ + p'_+) \int d^4x\, \rho_A(x_+, x_T) e^{ix\cdot(p'-p)} \\
&= it_{\rm forw}\ c(p_+, p'_+) A \int d^2x_T\, T_A(x_T) e^{-ix_T\cdot(p'_T - p_T)}.
\end{aligned}
$$

$\rho_A(x_+, x_T)$ is the nuclear density normalized to 1,

$$
T_A(x_T) = \int_{-\infty}^{+\infty} dx_+\, \rho_A(x_+, x_T)
$$

the nuclear profile, $|x_T| = b$ the impact parameter and $c(p_+, p'_+) = (2\pi) 2p_+ \delta(p'_+ - p_+)$ a normalization factor.

a) Show that the corresponding cross section can be written as an incoherent superposition of the contribution from the A scattering centers.

b) Extend this result to two scatterings (Fig. 1 right), performing the integral over k_- using the Cauchy theorem to get

$$
\begin{aligned}
& c(p_+, p'_+) i\mathcal{T}_2(q) = iA(A-1)(it_{\rm forw})^2 \int \frac{d^4k}{(2\pi)^4} d^4x_1 d^4x_2\, e^{ix_1\cdot(k-p)} \\
& \times e^{ix_2\cdot(p'-k)} \frac{(p_+ + k_+)(k_+ + p'_+)}{k^2 + i\epsilon} \rho_A(x_{1+}, x_{1T}) \rho_A(x_{2+}, x_{2T}) \\
& = c(p_+, p'_+) A(A-1)(it_{\rm forw})^2 \qquad (0.1) \\
& \times \int \frac{d^2k_T}{(2\pi)^2} dx_{1+} dx_{2+} d^2x_{1T} d^2x_{2T}\, e^{-ik_T^2(x_{2+} - x_{1+})/(2p_+)} \\
& \times e^{-i[x_{1T}\cdot(k_T - p_T) + x_{2T}\cdot(p'_T - k_T)]} \rho_A(x_{1+}, x_{1T}) \rho_A(x_{2+}, x_{2T}) \theta(x_{2+} - x_{1+}),
\end{aligned}
$$

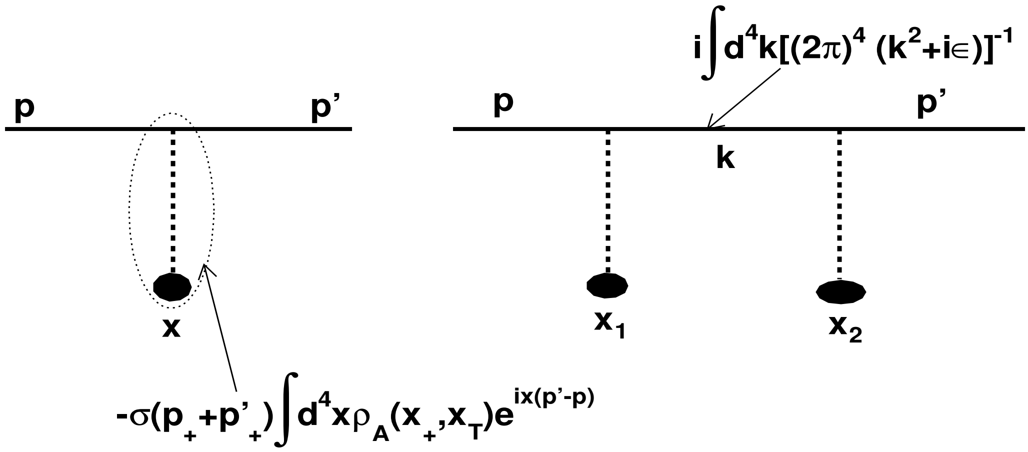

Figure 1: One- (left) and two- (right) scattering diagrams, with the corresponding Feynman rules written on them. Note that in the plot, σ should read $\sigma/2$.

with $\theta(x)$ the step function.
c) Analyze the low- and high-energy behavior of the exponential involving the difference in longitudinal position of the scattering centers, an interference term which makes this two-scattering contribution negligible at low energies and coherent at high energies. Show that in the totally coherent limit becomes negative - shadowing.
Note: In the coherent limit, the contribution of an arbitrary number of scatterings can be resummed, resulting in a path-ordered exponential for the $\mathcal{S}$-matrix. For QCD, this is the Wilson line.
References: N. Armesto, J. Phys. G **32** (2006) R367 [hep-ph/0604108], Section 2; A. Hebecker, A. Hebecker, Phys. Rept. **331** (2000) 1 [hep-ph/9905226], Section 3.1 and Appendix A.

2. The $x - Q^2$ kinematic plane

From the four momenta of the electron and hadron, determine the kinematic plane $x - Q^2$ for DIS at different accelerators HERA (E_e = 30 GeV, E_p = 900 GeV), EIC (E_e = 10 GeV, E_p = 100 GeV) and LHeC (E_e = 60 GeV, E_p = 7000 GeV). Compare it with the equivalent $x - p_\perp^2/M^2$ for particle production in a hadronic collider, e.g. the LHC, using $2 \to 2$ kinematics.
References: R. Devenish and A. Cooper-Sarkar, "Deep inelastic scattering," Oxford, UK: Univ. Pr. (2004) 403 p, Subsection 5.2; J. L. Abelleira Fernandez *et al.* [LHeC Study Group], J. Phys. G **39** (2012) 075001 [arXiv:1206.2913 [physics.acc-ph]], Subsection 4.1.3 and Section 11.2; J. L. Albacete and C. Marquet, Phys. Rev. Lett. **105** (2010) 162301 [arXiv:1005.4065 [hep-ph]].