Theory of the Deconfinement Transition and its Signatures
- Lecture 3 -

Chihiro Sasaki
Institute of Theoretical Physics
University of Wroclaw

Outline

Lecture 1: Brief introduction

Lecture 2: Interacting quarks and gluons

- ❑Polyakov-loop model: deconfinement
- ❑Nambu--Jona-Lasinio model: chiral SB

Lecture 3: Critical behaviors

- ❑Phase transition and the Landau theory
- ❑Fluctuations of conserved charges

QCD with Nf=2,3

- SU(Nf)_L x SU(Nf)_R chiral symmetry
  - Vanishing quark mass: exact symmetry
  - Finite quark mass: approximate but still reliable
  - Quark condensate <qqbar>: order parameter
- Hot/dense QCD
  - QCD partition function

$$Z = \mathrm{tr}\left[e^{-(H_{\mathrm{QCD}}(m_q=0)+\bar{q}(m_q-\mu_q\gamma_0)q)/T}\right] = e^{P(T,\mu)V/T}$$

  - Quark condensate

$$\langle\bar{q}q\rangle = -\frac{\partial P(T,\mu)}{\partial m_q}$$

[Details of low-T calc. see Gerber & Leutwyler ('89)]

Dropping quark-condensate

- High-temperature sector: P(T) = Pq(T) + Pg(T)

$$P_{\rm quark}(T; m_q) = 4N_c \int \frac{d^3k}{(2\pi)^3} T \ln\left(1 + e^{-E_q(k)/T}\right)$$

$$\stackrel{m_q/T \ll 1}{\simeq} \frac{7N_c}{2}\left[\frac{\pi^2}{90}T^4 - \frac{1}{42}m_q^2 T^2 - \frac{m_q^4}{56\pi^2}\left(\ln\left(\frac{m_q^2}{(\pi T)^2} + C\right)\right) + \cdots\right]$$

→ <qqbar> = 0 at T=∞ and mq = 0.

- Low-temperature sector: P(T) = P_π (T)

$$\frac{\langle \bar{q}q\rangle_{\rm med}}{\langle \bar{q}q\rangle_{\rm vac}} \stackrel{\text{GOR}}{=} 1 + \frac{1}{f_\pi^2}\frac{\partial P_\pi(T)}{\partial m_\pi^2}$$

$$= 1 - \frac{T^2}{8f_\pi^2} - \frac{1}{6}\left(\frac{T^2}{8f_\pi^2}\right)^2 - \frac{16}{9}\left(\frac{T^2}{8f_\pi^2}\right)^3 \ln\left(\frac{\Lambda_q}{T}\right) + \mathcal{O}(T^8)$$

Condensate decreases with T!

Chiral phase transition

- Sketch: Nf=2 NJL model at finite T

$$V(\sigma) = \frac{\sigma^2}{4G} - \frac{1}{16\pi^2}\left[(\Lambda^4 - \sigma^4)\ln\left(1 + \frac{\sigma^2}{\Lambda^2}\right) + \sigma^2\Lambda^2\right]$$

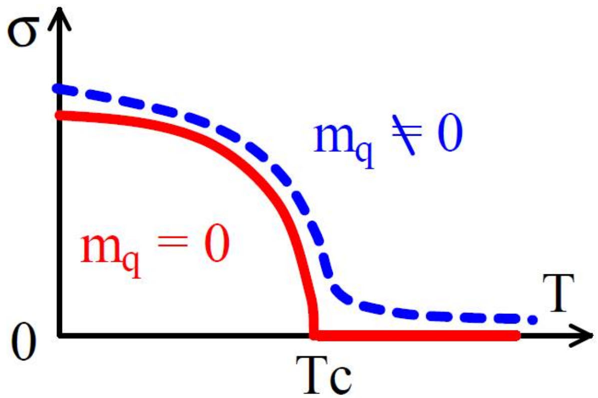
$$\Omega = V(\sigma) + 2\gamma_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - n(p,T)\right)$$

γ_q: $\text{spin} \times \text{flavor} \times \text{color} = 12$

$n(p,T)$: $(1+\exp[E/T])^{-1}$

- 2^{nd} order transition if mq=0

$$T_c^2 = \frac{1}{2G_{cr}} - \frac{1}{2G}$$



1. Landau theory

General theory for a phase transition

- Partition function in the thermodynamic limit

$$Z = e^{-\Omega(K)} = e^{P(K)V}$$

  - 1/T absorbed in the def. of Ω and P
  - K: a set of parameters, e.g. T, μ , g, external fields

- Order of the phase transition

$$\frac{\partial P(K)}{\partial K} \begin{cases} \text{discontinuous} & \text{1st order} \\ \text{continuous} & \text{2nd order} \end{cases}$$

General theory for a phase transition

Landau potential of an order parameter σ (x)

$$Z = \int \mathcal{D}\sigma \, e^{-S_{\rm eff}(\sigma(x);K)}$$

- ❑ Mean field approximation
  - The integral is dominated by a minimum of S_eff.
  - Fluctuations around the minimum are neglected.
  - Uniform system: σ as x-indep. order parameter

  → S_eff = V_eff(σ ;K)V
- ❑ Expanding V_eff into a power series of σ ;

$$V_{\rm eff}(\sigma;K) = \sum_n a_n(K)\sigma^n$$

[Table adapted from Yagi, Hatsuda and Miake, “Quark-Gluon Plasma”]

V_{eff}	spin system in $d = 3$	QCD
2nd order $\frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 - h\sigma$ controlled by (a, h)	Ising model $\sigma \sim M$ Magnetization $(a, h) \leftrightarrow (T, H)$ ↑ External magnetic field	$N_c = 3, N_f = 2$ $\sigma \sim \langle \bar{u}u + \bar{d}d \rangle$ $(a, h) \leftrightarrow (T, m_q)$ $N_c = 2, N_f = 0$ $\sigma \sim \langle \Phi \rangle$ $(a, h) \leftrightarrow (T, 1/m_Q)$
1st order $\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4 - h\sigma$ controlled by (a, h)	Potts model $\sigma \sim M$ $(a, h) \leftrightarrow (T, H)$	$N_c = 3, N_f = 3$ $\sigma \sim \langle \bar{u}u + \bar{d}d + \bar{s}s \rangle$ $(a, h) \leftrightarrow (T, m_q)$ $N_c = 3, N_f = 0$ $\sigma \sim \langle \Phi \rangle$ $(a, h) \leftrightarrow (T, 1/m_Q)$

2^{nd} order phase transition

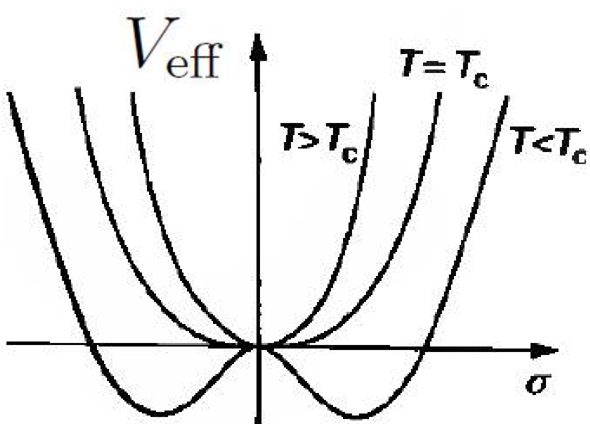
- Landau potential truncated at n=4 & w/o n=3

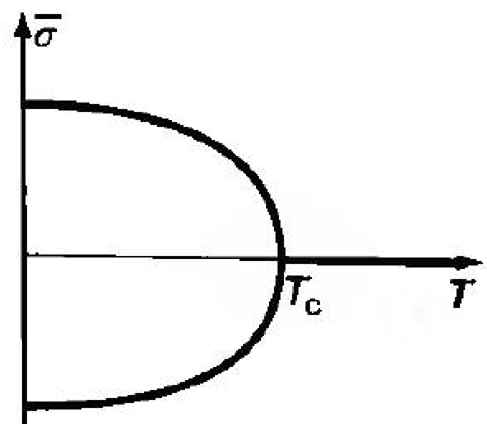
$$V_{\rm eff} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 - h\sigma$$

$$a = a_t t = a_t(T-T_c)/T_c\,,\ a_t > 0\,,\ b > 0\,,\ h \geq 0$$

- Stationary condition

$$\frac{\partial V_{\rm eff}}{\partial \sigma} = a\sigma + b\sigma^3 - h = 0 \ \Rightarrow\ \bar{\sigma}|_{h=0} = \begin{cases} 0 & (T \geq T_c) \\ \pm(-a/b)^{1/2} & (T < T_c) \end{cases}$$





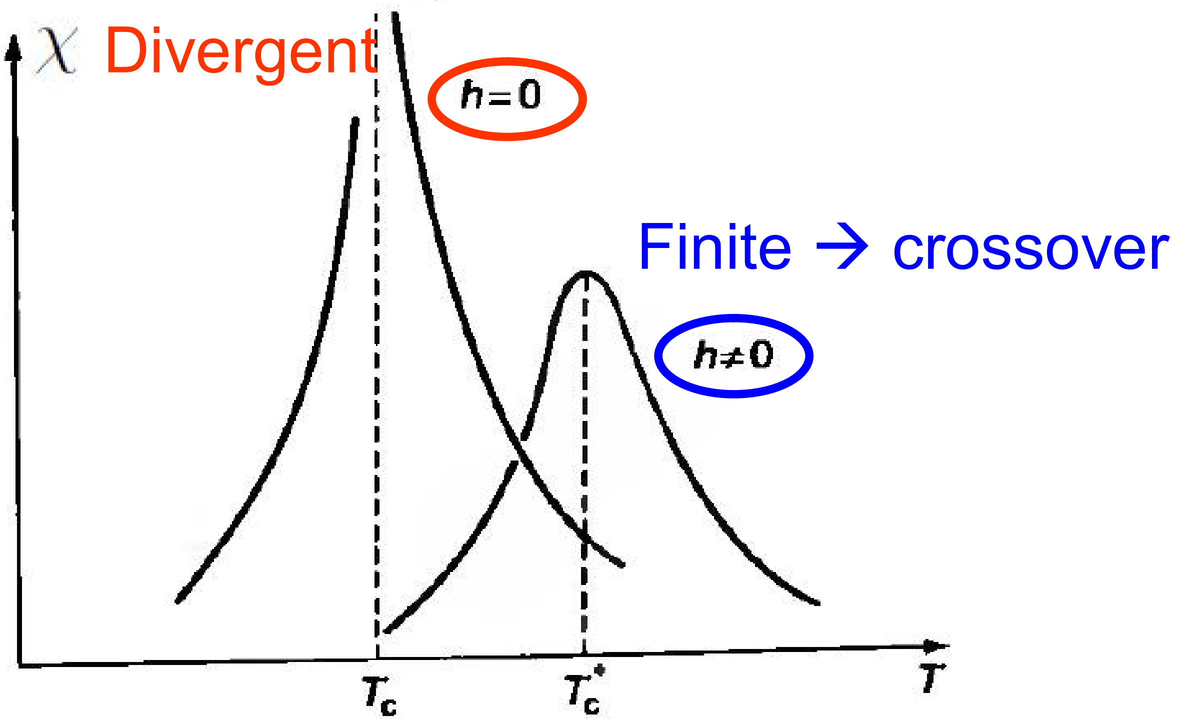
h≠0 → crossover

$$\bar{\sigma}(T=T_c) = (h/b)^{1/3}$$

2nd order phase transition

- Susceptibility of the order parameter

$$\chi(T,h)|_{h=0} = \frac{\partial\bar{\sigma}}{\partial h}|_{h=0} = \begin{cases} 1/a \sim |t|^{-1} & (T \geq T_c) \\ 1/(-2a) \sim |t|^{-1} & (T < T_c) \end{cases}$$



Critical exponents

- Quantities sensitive to the phase transition

$$\bar{\sigma}(T \to T_c^-, h=0) \sim |t|^{\beta},$$

$$C_V(T \to T_c^{\pm}, h=0) = -T\frac{\partial^2 V_{\rm eff}}{\partial T^2}|_{h=0} \sim |t|^{-\alpha_\pm},$$

$$\bar{\sigma}(T = T_c, h \to 0) \sim h^{1/\delta},$$

$$\chi(T \to T_c^{\pm}, h=0) \sim |t|^{-\gamma_\pm}$$

- Mean field theory

$$\alpha_\pm = 0\,,\ \ \beta = 1/2\,,\ \ \gamma_\pm = 1\,,\ \ \delta = 3$$

Critical exponents

❑ Mean field theory

- Independent of details of underlying dynamics
- Crucial conditions: no cubic term & positive b

❑ Beyond the Landau theory

- Fluctuations around the mean field
  - ➢ 2nd order may become either 1st order or crossover
  - ➢ 2nd order: different critical exponents from MF

    O(4): $\alpha = -0.24$, $\beta = 0.38$, $\gamma = 1.4$, $\delta = 4.7$

    vs. MF: $\alpha = 0$, $\beta = ½$, $\gamma = 1$, $\delta = 3$
- Spatial dimensions, internal symmetry

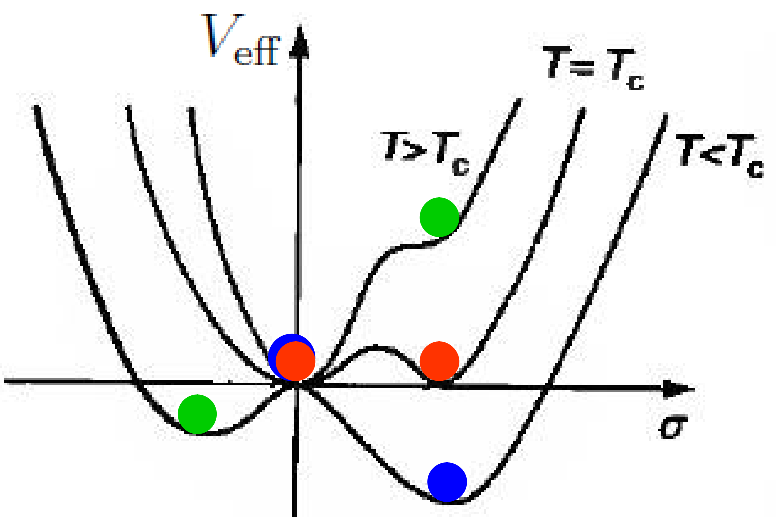
1st order phase transition

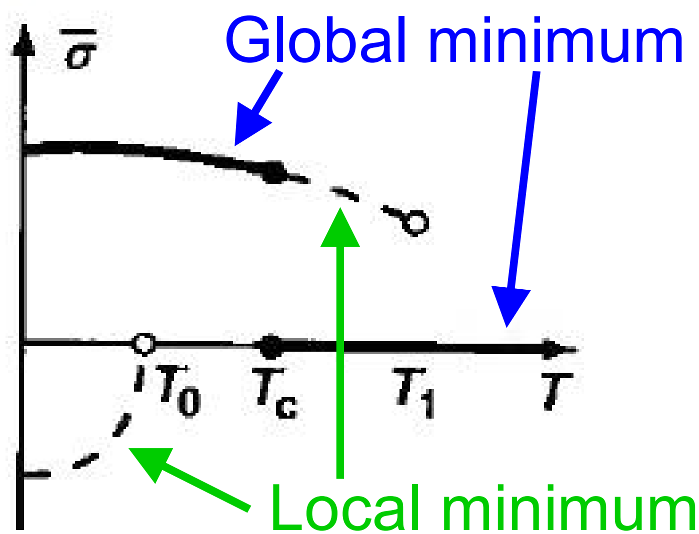
- Landau potential truncated at n=4

$$V_{\text{eff}} = \frac{1}{2}a\sigma^2 - \frac{1}{3}c\sigma^3 + \frac{1}{4}b\sigma^4 - h\sigma$$

$$a = a_t t = a_t(T - T_0)/T_0\,,\;\; a_t > 0\,,\;\; b > 0\,,\;\; c > 0\,,\;\; h \geq 0$$

h=0 → $\bar{\sigma} = 0\,,\;\; \dfrac{c \pm \sqrt{c^2 - 4ab}}{2b}$





Tricritical point

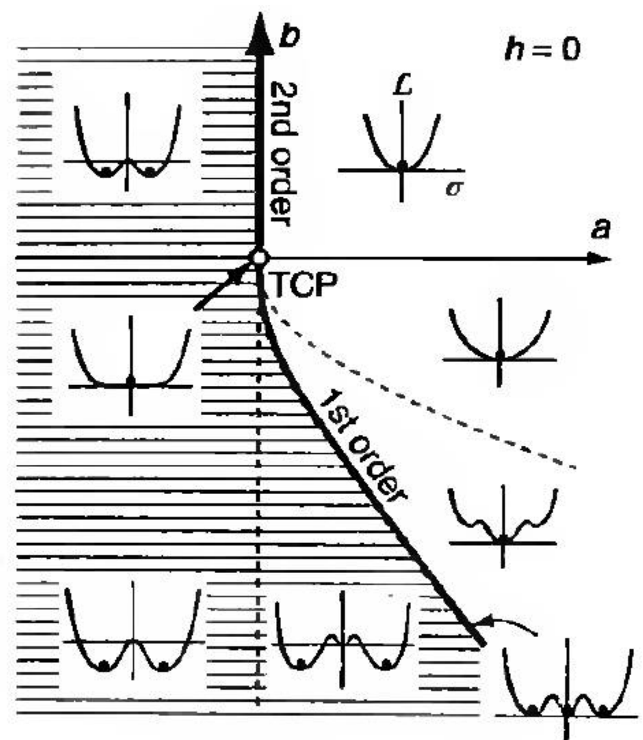
- Landau potential truncated at n=6 w/o n=3,5

$$V_{\rm eff} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 + \frac{1}{6}c\sigma^6 \cancel{- h\sigma}\,, \quad c > 0$$

$$a = a_t t + a_s s\,,\ \ b = b_t t + b_s s\,,\ \ t = (T - T_c)/Tc\,,\ \ s = (\mu - \mu_c)/\mu_c$$

$$\bar{\sigma} = 0\,,\quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$$

- TCP: a = b = 0
- 2^{nd} order: b > 0, a = 0
- 1^{st} order: b < 0, a = $3b^2/16c$
- Metastable: a = 0, b < 0 and a = $b^2/4c$, b < 0

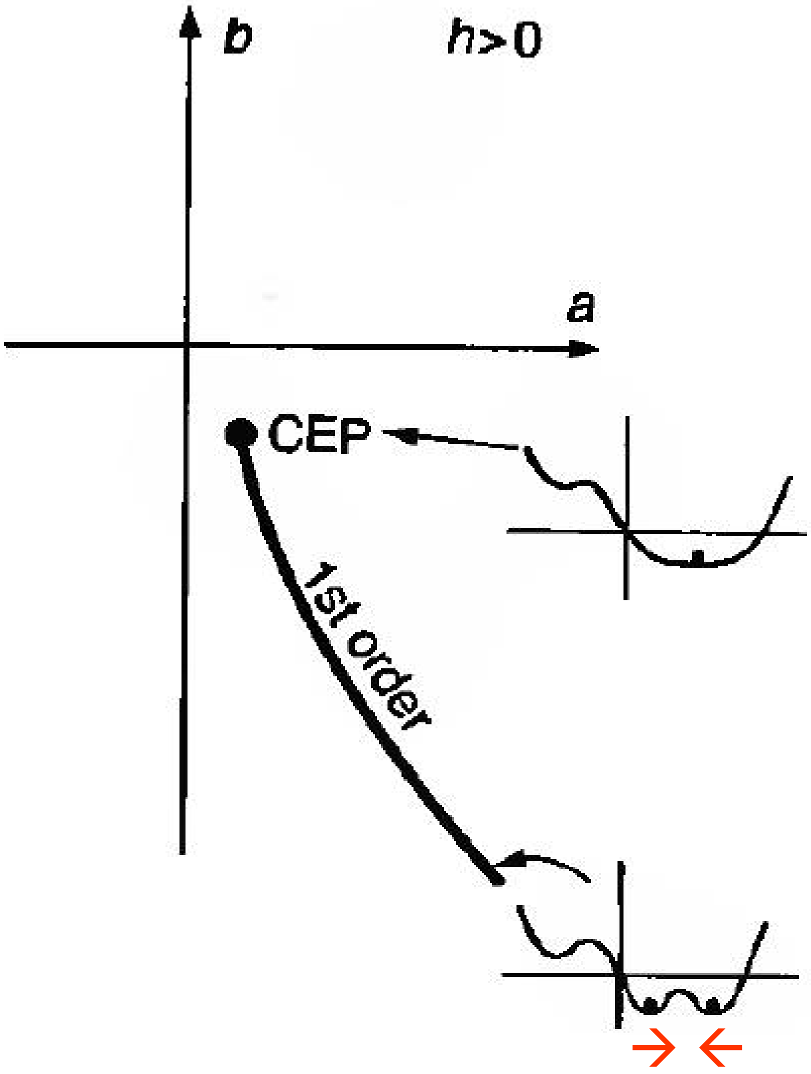


Critical (end) point

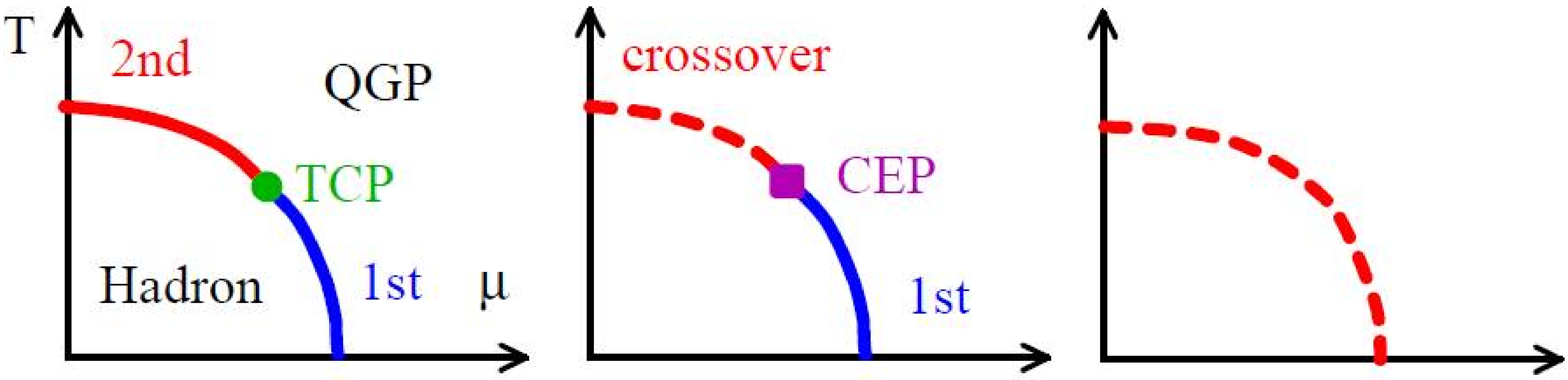
- When h≠0,
  - 2nd order line disappear.
  - 1st order line shifts.
- CEP:

  flat effective-potential

$$\frac{\partial^n V_{\rm eff}}{\partial \sigma^n} = 0 \ (n = 1, 2, 3)$$



QCD phase diagram



- Location of TCP/CEP: not established
- 1st order at large μ :not established
- No CEP as an option of QCD
- (much more?) complicated phase diagram

[For more details, Fukushima and CS (2013)]

2. Fluctuations of conserved charges

[Refs. Stephanov et al. (1999)]

Observables

Order parameter: <qqbar> **not observable!**

- Strategy
  - Quantity to which σ mode couples
  - Susceptibility as a good indicator of p.t.
- Conserved charges: X = {baryon number B, electric charge Q, strangeness S, etc.}
  - Generalized susceptibilities and their ratios

$$\chi_n^X = \frac{1}{VT^3} \frac{\partial^n \ln Z}{\partial(\mu_X/T)^n} \qquad R_{n,m}^X = \frac{\chi_n^X}{\chi_m^X}$$

No volume factor!

[Refs. Ejiri et al. (2006), Skokov et al. (2010)]

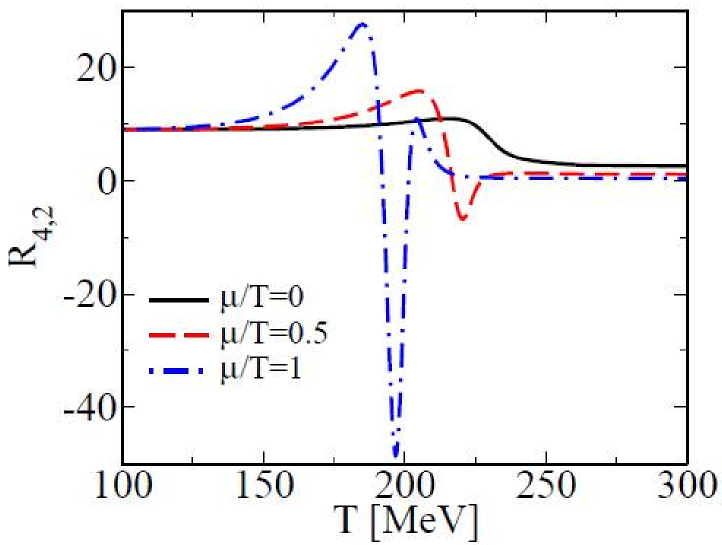
Signatures of deconfinement

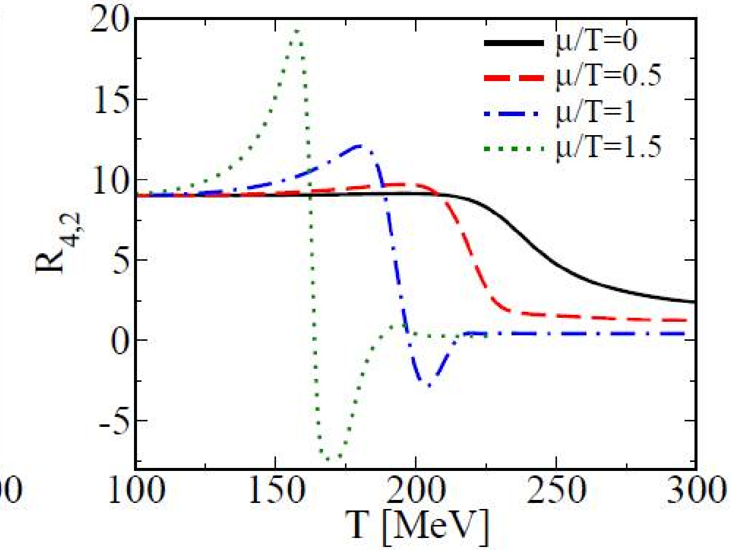
- Sensitive to chiral transition: O(4) criticality
  - At $\mu = 0$: χ (B,n) for even n > 4
  - At $\mu \neq 0$: χ (B,n) for n > 2

$$\mu_i = \mu_B B_i$$

- Sensitive to deconf. transition: R(4,2) w/ X=B
  - At low T: R = (3B)^2 = 9 ← Boltzmann approx.
  - At high T: R = (3B)^2 = 1 ← Stefan-Boltzmann

MF

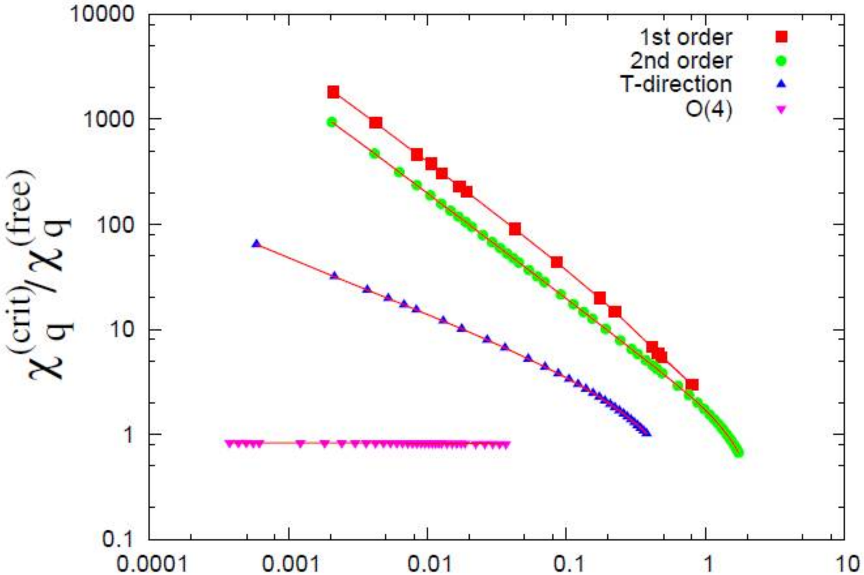




FRG

[Refs. Halasz et al. (‘98), Berges and Rajagopal (‘99)]

Signatures of QCD CEP

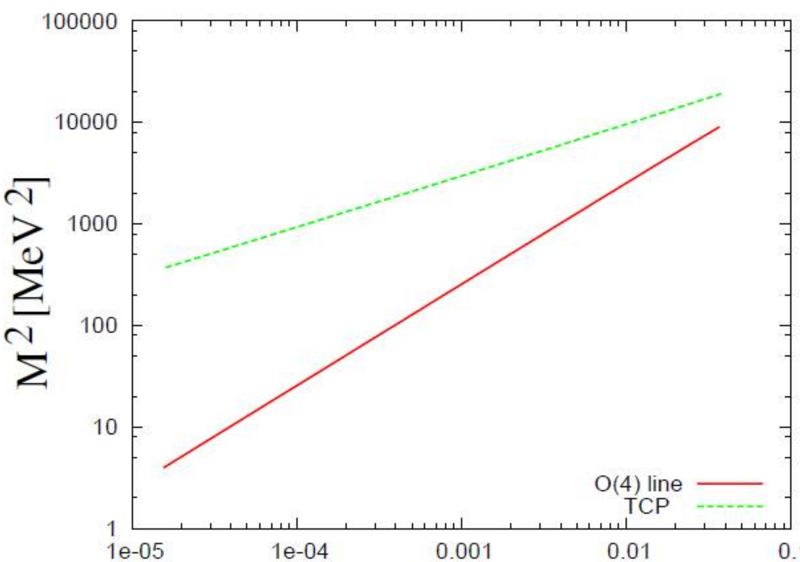


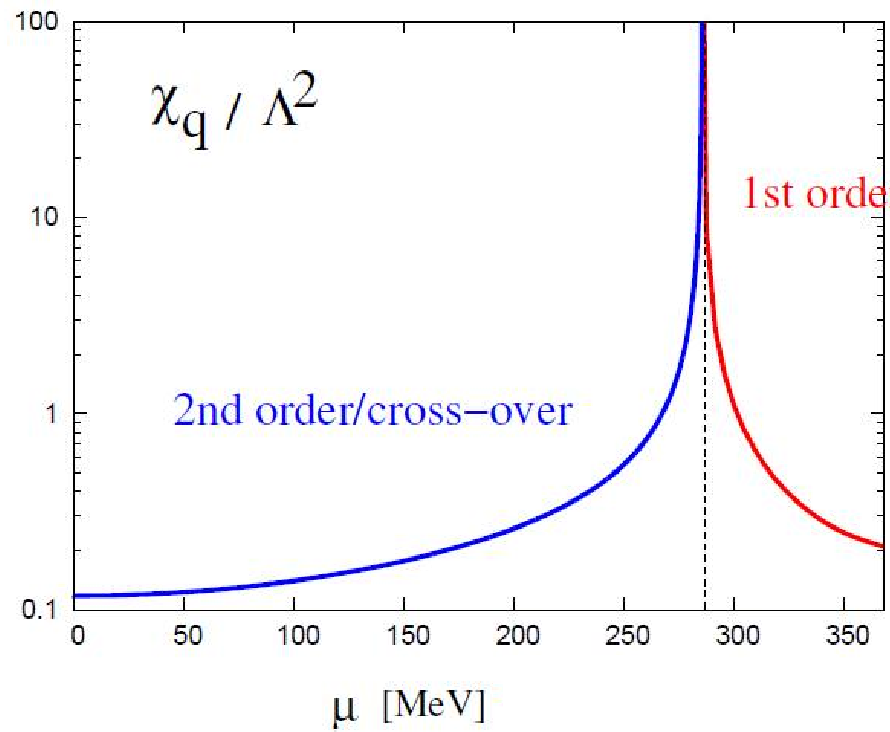
✓Critical exponent changes.

✓Landau theory $\chi_B \sim M^2 \chi_{\rm ch}$

✓Effective models, SD eq.

[Figs: illustrations w/ MF NJL]





Summary

- ***Schematic*** models --- how much useful?
  - Intuitive, systematic, guided by symmetries
  - Capture the essential physics
- You must ask the ***right*** question!
  - Case study under possible options/conditions
  - Extract characteristic properties
  - Do not quantify e.g. CEP location!
- Towards more ***realistic*** situations: dynamics, finite volume, rapidity

Good baseline!