Jet suppression and energy loss

Konrad Tywoniuk

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OUTLINE

1) Jet physics in vacuum

2) Interactions with the medium

3) Energy loss

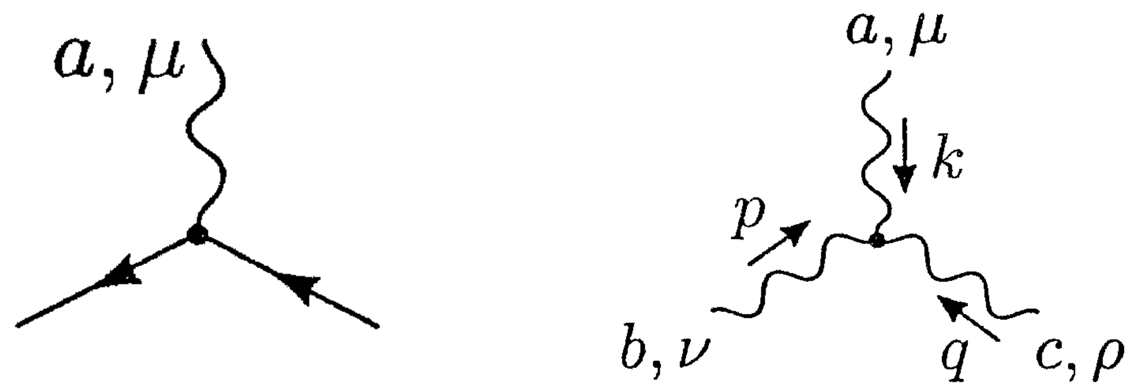
1) Jet physics in vacuum

- basics
- parton splitting
- angular ordering
- leading-logarithmic behavior

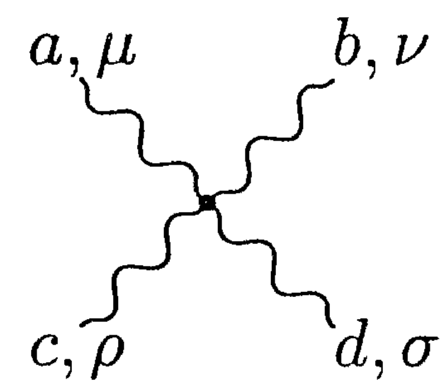
QCD: QUARKS AND GLUONS

Theory of quarks (fermions) and gluons (bosons) and their interactions.

Work in physical gauge (no ghosts).

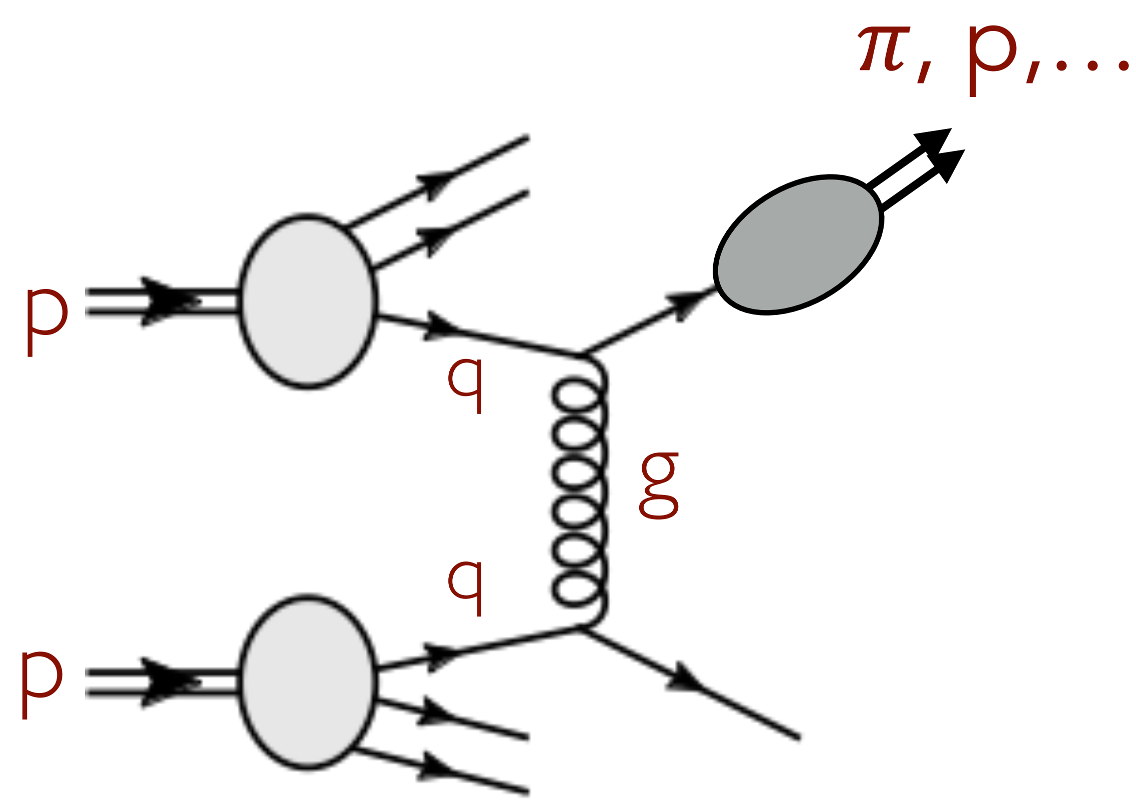


Most important for the physics of this lecture…



- keep in mind: we observe hadrons!
- quarks and gluons are DOF's in perturbation theory!

QCD FACTORIZATION



- jet/hadron production
- separation of processes
  - short-distance (perturbative)
  - long-distance (non-perturbative)
- hard matrix element
- corrections suppressed ~$1/Q^2$

$$\sigma^{pp\to h} = f_p^i(x_1,Q^2)\otimes f_p^j(x_2,Q^2) \otimes \hat{\sigma}^{ij\to k} \otimes D^{k\to h}(z,Q^2)$$

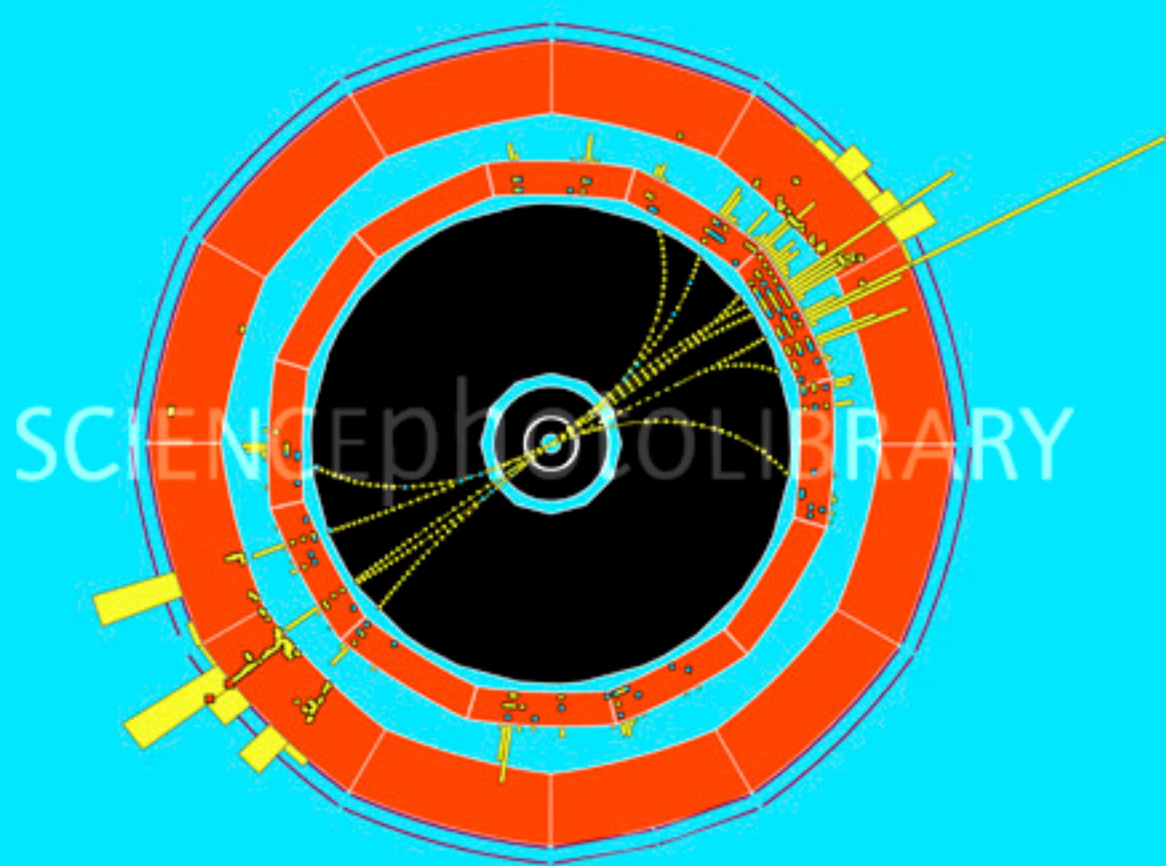
parton distribution functions

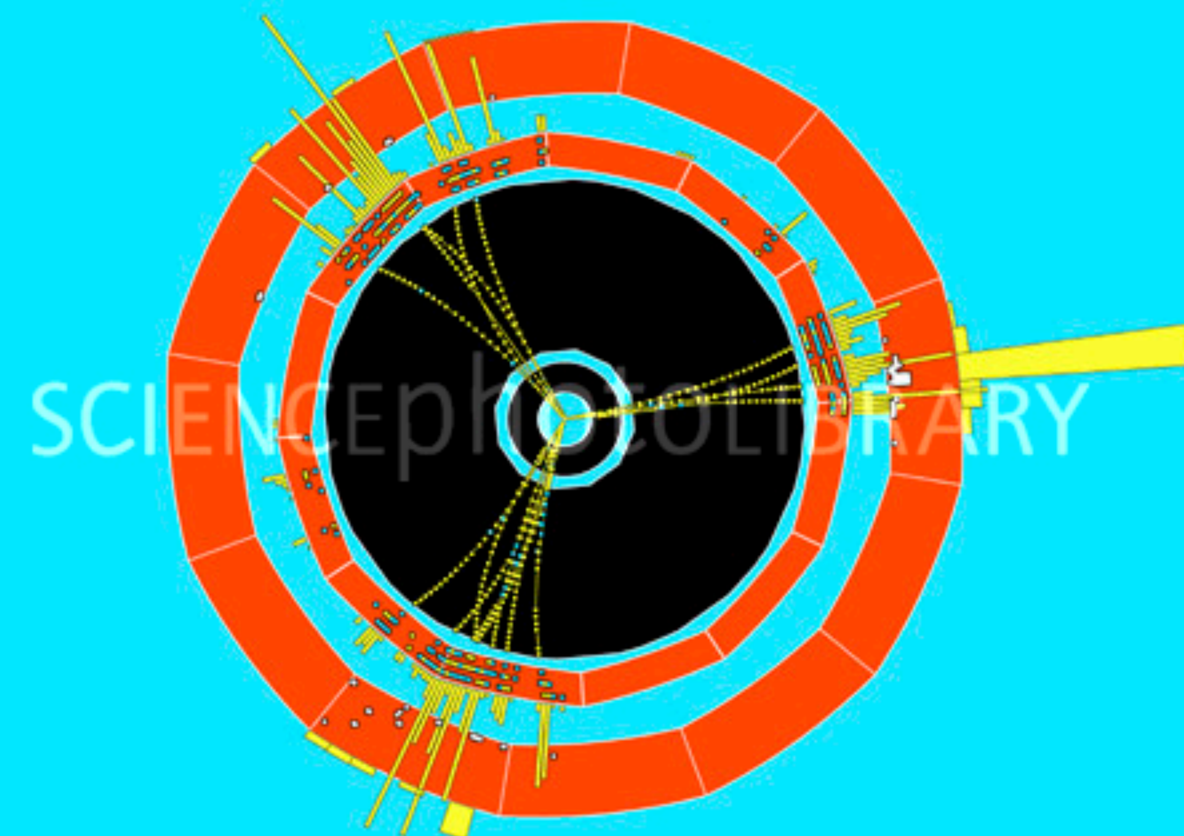
fragmentation function

These lectures will deal with the final-state evolution

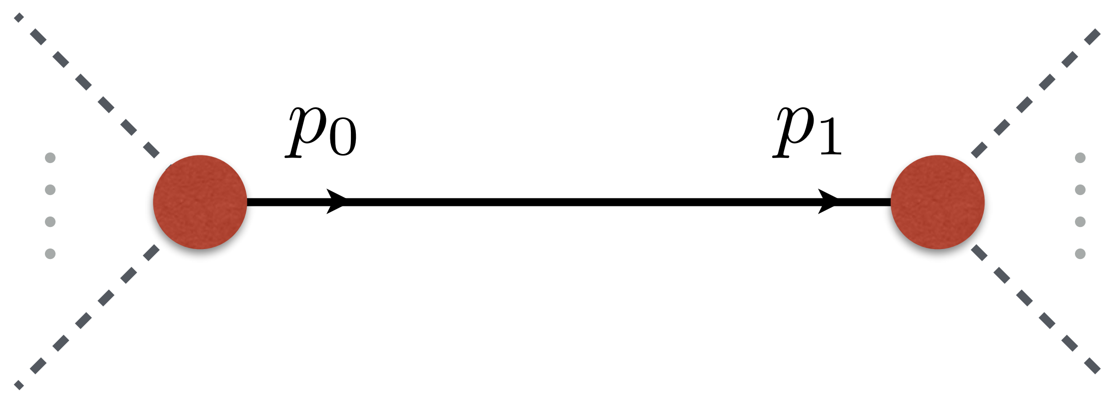
(tomorrow: interactions, etc…)

How can we make sense of these “images”?





Gluon propagator



$$\mathcal{M}(\{p_{\rm out}\},\{p_{\rm in}\}) = \int \frac{{\rm d}^4 p_1}{(2\pi)^4}\frac{{\rm d}^4 p_0}{(2\pi)^4}(2\pi)^4\delta(p_{\rm in}-p_0)(2\pi)^4\delta(p_1-p_{\rm out})$$
$$\times\, V_\mu(\{p_{\rm out}\},p_1)\, G^{\mu\nu}(p_1,p_0)\, V_\nu(p_0,\{p_{\rm in}\})$$

- how do particles move from one vertex to another: propagator
- our example: gluons (quarks are completely analogous)
- play with momentum conservation: trade for **time** information

$$(2\pi)^4\delta(p_{\rm in}-p_0) = (2\pi)^3\delta(\vec{p}_{\rm in}-\vec{p}_0)\int_{-\infty}^{\infty}{\rm d}t_0\,{\rm e}^{-i(p^-_{\rm in}-p^-_0)t_0}$$

A NOTE ON NOTATION

Light-cone momenta/coordinates:

$$p^+ = \frac{1}{2}(p^0 + p^3)$$
$$p^- = p^0 - p^3$$

$$\begin{aligned} p = & \ (p^0, \boldsymbol{p}, p^3) \\ = & \ (p^+, p^-, \boldsymbol{p}) \end{aligned}$$

Scalar product becomes: $p \cdot k = p^+ k^- + p^- k^+ - \boldsymbol{p} \cdot \boldsymbol{k}$

Boost-invariant PS element: $\mathrm{d}\Omega_p \equiv \frac{\mathrm{d}k^+ \, \mathrm{d}^2\boldsymbol{k}}{(2\pi)^3 \, 2p^+}$

Shorthand:

$$t \leftrightarrow x^+$$
$$E \leftrightarrow p^+$$

Vacuum propagator

$$G^{\mu\nu}(p_1, p_0) = (2\pi)^4 \delta(p_1 - p_0)\, d^{\mu\nu}(p) \frac{-i}{p^2 + i\epsilon}$$

Light-cone (physical) gauge: $n \cdot A = A^- = 0$

$$d^{\mu\nu}(p) = \sum_\lambda (-1)^{\lambda+1} \varepsilon_\lambda^\mu(p) \varepsilon_\lambda^{*,\nu}(p) = g^{\mu\nu} - \frac{p^\mu n^\nu + p^\nu n^\mu}{p \cdot n}$$

$$= \underbrace{d^{\mu\nu}(p) + d^{\mu i}(p) d^{i\nu}(p)}_{d_L^{\mu\nu}(p)} + d_T^{\mu\nu}(p)$$

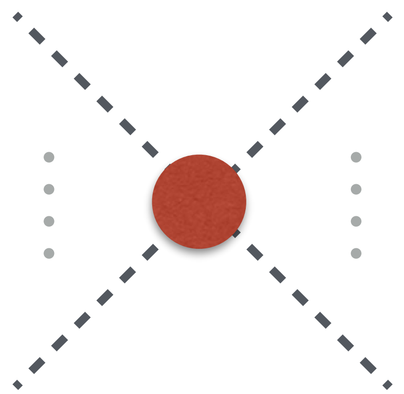
Non-vanishing components:

$$d_L^{--}(p) = -\frac{p^2}{(p^+)^2}$$

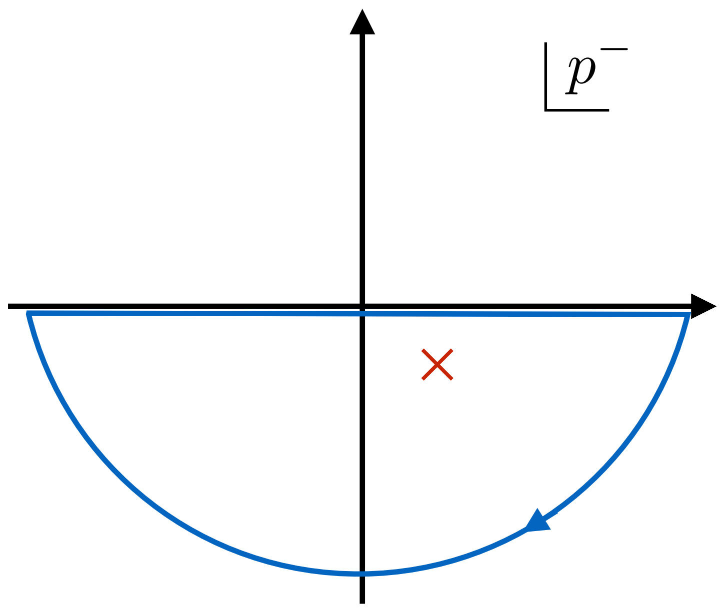
$$d_T^{--}(p) = -\frac{\boldsymbol{p}^2}{(p)^+}\,, \qquad d_T^{-i}(p) = -\frac{p^i}{p^+}\,, \qquad d_T^{ij}(p) = -\delta^{ij}\,.$$

Longitudinal modes

$$\int \frac{\mathrm{d}p^-}{2\pi}\, \mathrm{e}^{-ip^-(t_1-t_0)} = \delta(t_1 - t_0)$$



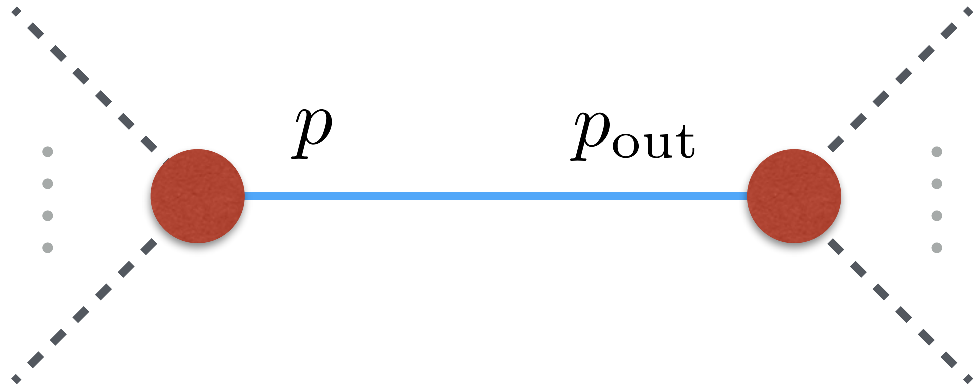
- non-propagating (instantaneous) modes
- absorbed into higher-order vertices (e.g. 4-point vertex)



Transverse modes

- retarded propagator
- iϵ prescription ("adiabatic turn-off")

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}p^-}{2\pi} \frac{-i\mathrm{e}^{-ip^-(t_1-t_0)}}{p^2 + i\epsilon} = \frac{-1}{2p^+}\Theta(p^+)\Theta(t_1 - t_0)\mathrm{e}^{-i\left[\frac{\boldsymbol{p}^2}{2p^+} - i\epsilon\right](t_1-t_0)}$$



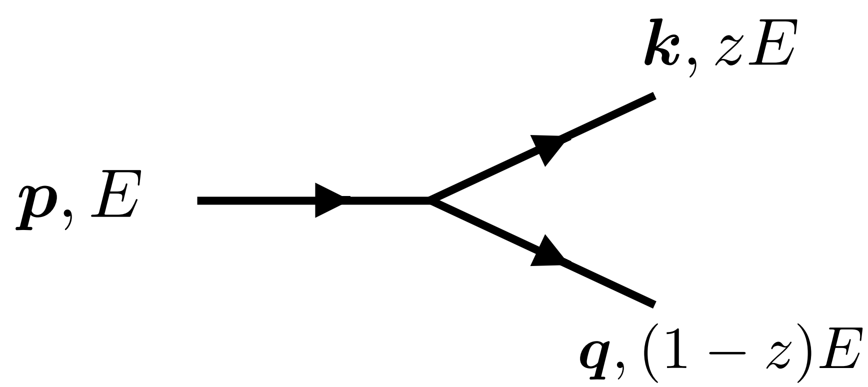
$$\mathcal{M}(\{p_{\rm out}\},\{p\}) = (2\pi)^3\delta(\vec{p}-\vec{p}_{\rm out})\int_{-\infty}^{\infty}{\rm d}t_0\int_{t_0}^{\infty}{\rm d}t_1$$

$$\times\, {\rm e}^{ip^-_{\rm out}t_1}\Gamma^i(\{p_{\rm out}\},p)\,\frac{1}{2p^+}{\rm e}^{-i\frac{\boldsymbol{p}^2}{2p^+}(t_1-t_0)}\,\Gamma^i(p,\{p_{\rm in}\}){\rm e}^{-ip^-t_0}$$

$$\Gamma^i(\{p_{\rm out}\},p) = V_\mu(\{p_{\rm out}\},p)d^{\mu i}(p)$$

- we are left with "effective" vertices (sandwiched between projection operators)
  - explicit time dependence
  - hard vertex/short times: allow $t_0 \to 0$ in the propagator
- propagator in mixed (transverse momentum & time) representation

VERTEX STRUCTURE



$$= g f^{abc}\big[g^{\nu\rho}(-k-p)^{\mu} + g^{\mu\rho}(p+q)^{\nu} + g^{\mu\nu}(k-q)^{\rho}\big]$$

$$\Gamma^{ijl}(\boldsymbol{k}-z\boldsymbol{p}, z) = d^{j\nu}(k)d^{i\mu}(p-k)V^{abc}_{\mu\nu\rho}(k-p,-k,p)d^{\rho l}(p)$$

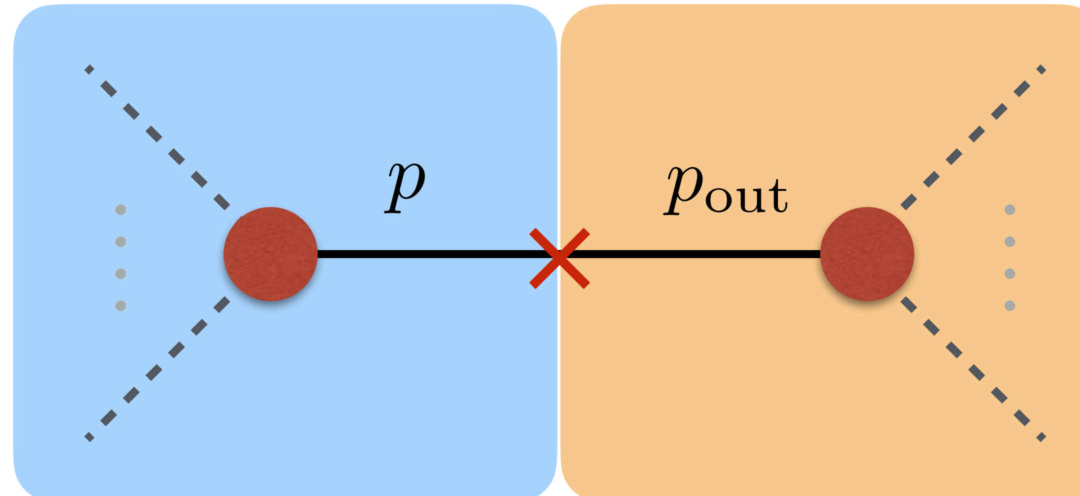
$$= 2gf^{abc}\left[\frac{1}{1-z}(\boldsymbol{k}-z\boldsymbol{p})^{i}\delta^{jl} + \frac{1}{z}(\boldsymbol{k}-z\boldsymbol{p})^{j}\delta^{il} - (\boldsymbol{k}-z\boldsymbol{p})^{l}\delta^{ij}\right]$$

- vertex depends only on $\boldsymbol{k}_{\rm rel} = \boldsymbol{k} - z\boldsymbol{p}$
- rotational (Galilean) symmetry
- similar procedure for quark splitting (dress with relevant spinors)

CUT PROPAGATOR

$$\frac{-i}{p^2 + i\epsilon} \to 2\pi\delta(p^2)$$

- no time ordering
- no $i\epsilon$ prescription



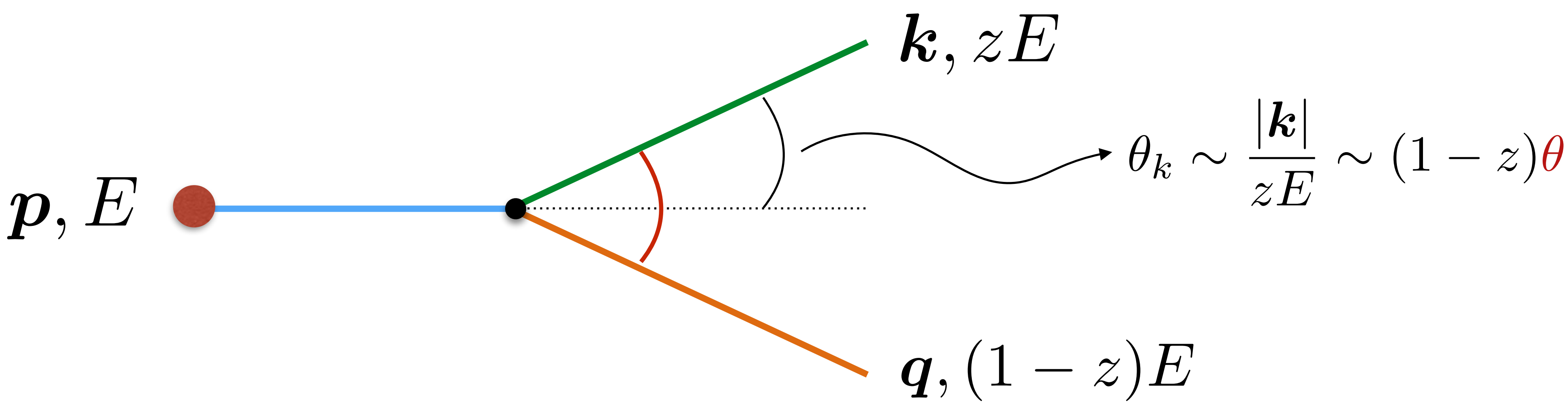
$$\mathcal{M}(\{p_{\rm out}\},\{p\}) = \frac{1}{2p^+}(2\pi)^3\delta(\vec{p}-\vec{p}_{\rm out})\int_{-\infty}^{\infty} {\rm d}t_1\, {\rm e}^{-i\left(\frac{\boldsymbol{p}^2}{2p^+}-p^-_{\rm out}\right)t_1}\Gamma^i(\{p_{\rm out}\},p)$$

$$\times \int_{-\infty}^{\infty} {\rm d}t_0\, {\rm e}^{i\left(\frac{\boldsymbol{p}^2}{2p^+}-p^-\right)t_0}\Gamma^i(p,\{p_{\rm in}\})$$

$$= \frac{1}{2p^+}(2\pi)^3\delta(\vec{p}-\vec{p}_{\rm out})\mathcal{M}^* \times \mathcal{M}$$

4-momentum conservation is restored in the absence of further interactions!

Splitting amplitude



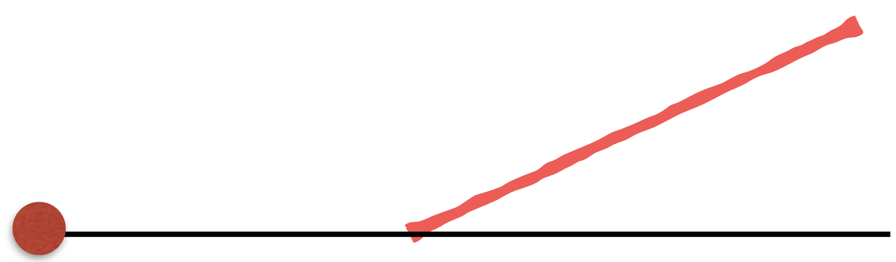
$$(q,k|\mathcal{M}_1|p) = (2\pi)^3\delta(\vec{p}-\vec{k}-\vec{q}) \int_0^\infty \mathrm{d}t\, \mathrm{e}^{i\frac{\boldsymbol{k}^2}{2zE}t}\mathrm{e}^{i\frac{(\boldsymbol{p}-\boldsymbol{k})^2}{2(1-z)E}t}\,\Gamma^{abc}_{ijk}(\boldsymbol{k}-z\boldsymbol{p},z)\,\frac{1}{2E}\mathrm{e}^{-i\left[\frac{\boldsymbol{p}^2}{2zE}-i\epsilon\right]t}$$

$$= \frac{1}{2E}(2\pi)^3\delta(\vec{p}-\vec{k}-\vec{q})\,\Gamma^{abc}_{ijk}(\boldsymbol{k}-z\boldsymbol{p},z)\int_0^\infty \mathrm{d}t\,\mathrm{e}^{i\frac{(\boldsymbol{k}-z\boldsymbol{p})^2}{2z(1-z)E}t}\mathrm{e}^{-\epsilon t}$$

Time-scale for splitting (formation time):

$$t_\mathrm{f} = \frac{2z(1-z)E}{\boldsymbol{k}^2_\mathrm{rel}} = \frac{2E}{m^2}$$

1-GLUON SPECTRUM



$$\frac{d\sigma}{d\Omega_k d\Omega_p} = \frac{1}{2(N_c^2-1)} \left|(k|\mathcal{M}_1|p)\right|^2 \frac{d\sigma}{d\Omega_p}$$

$$(k|\mathcal{M}_1|p) = i\frac{z(1-z)}{(\boldsymbol{k}-z\boldsymbol{p})^2}\Gamma^{abc}_{ijk}(\boldsymbol{k}-z\boldsymbol{p},z)$$

We will focus on the soft limit: $z \ll 1$

$$\Gamma_{ijl} \simeq \frac{2}{z} g f^{abc} \boldsymbol{k}^j_{\rm rel} \delta^{il} \quad \Rightarrow \quad \Gamma^2 \simeq \frac{8g^2}{z^2} N_c (N_c^2-1) \boldsymbol{k}^2_{\rm rel}$$

[For finite z, we would obtain the Altarelli-Parisi splitting function.]

Spectrum of emitted gluons: $$dN \simeq \frac{\alpha_s N_c}{\pi} \frac{dz}{z} \frac{d\boldsymbol{k}^2_{\rm rel}}{\boldsymbol{k}^2_{\rm rel}}$$

Proportional to strong coupling constant & color factor of emitter.

Spectrum is suppressed by α_s: why should we care?

$$\mathrm{d}N = 2\frac{\alpha_s N_c}{\pi}\,\frac{\mathrm{d}z}{z}\,\frac{\mathrm{d}\theta}{\theta}$$

Soft emission:

Collinear emission:

In both cases, the spectrum diverges!?

Perturbative condition: $k_\perp \leq Q_0 \sim \Lambda_{\rm QCD}$

$$N = \frac{\alpha_s C_F}{\pi} 2 \int_{Q_0/E}^{1} \frac{\mathrm{d}x}{x} \int_{Q_0/(xE)}^{1} \frac{\mathrm{d}\theta}{\theta} = \frac{\alpha_s C_F}{\pi} \log^2 \frac{E}{Q_0}$$

- smallness of the coupling constant compensated by large phase space
  - double-logarithmic approximation
  - improvements include single-log contributions

Intra-jet processes:

$$k_\perp \ll k^+ \ll p^+$$

$$N \sim \frac{\alpha}{\pi} \log^2 E \gtrsim 1$$

log resummations (N…LL)

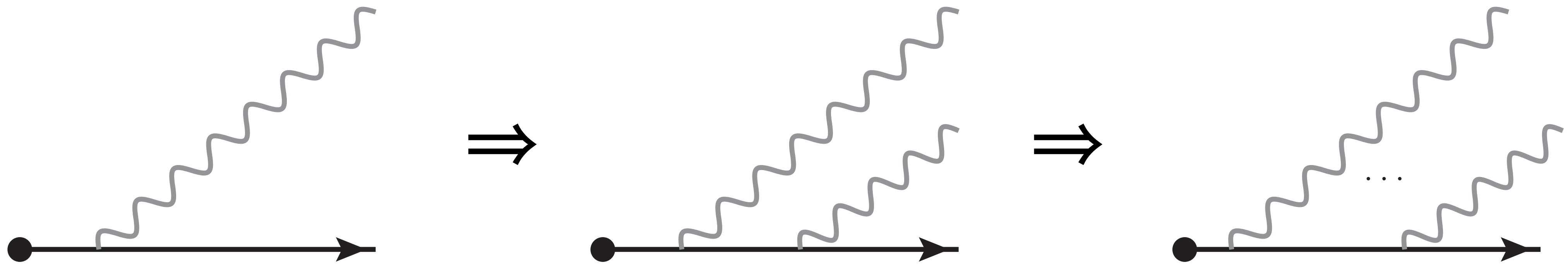
Inter-jet processes:

$$k_\perp \sim k^+ \sim p^+$$

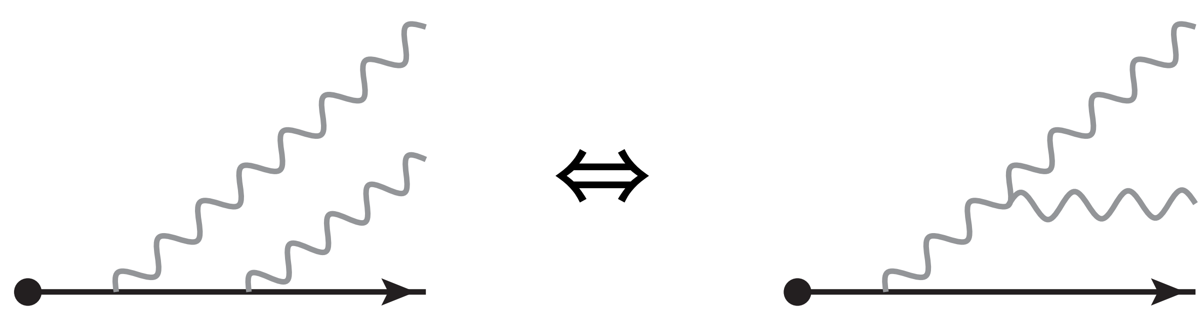
$$N \sim \frac{\alpha}{\pi} \ll 1$$

fixed-order (N…LO)

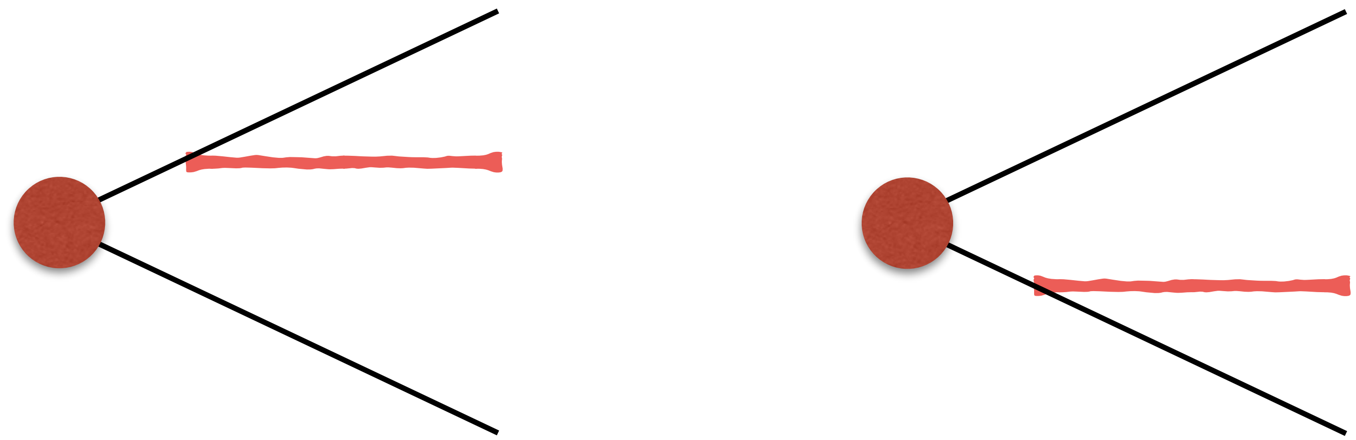
Multi-gluon emissions



- soft & collinear emissions: need to consider emissions of multiple gluons
- can we simply reiterate single-emission formula?
- for photons in QED: yes!
- for gluons in QCD: no! there are interferences!



ANTENNA SPECTRUM



$$(k|\mathcal{M}|p_1,p_2) = 2g\left[\mathcal{Q}_1\frac{\boldsymbol{k}-\omega\boldsymbol{n}_1}{(\boldsymbol{k}-\omega\boldsymbol{n}_1)^2} + \mathcal{Q}_2\frac{\boldsymbol{k}-\omega\boldsymbol{n}_2}{(\boldsymbol{k}-\omega\boldsymbol{n}_2)^2}\right]$$

$\boldsymbol{n}_i \equiv \boldsymbol{p}_i/E_i$

quark (color) charge $\mathcal{Q}_q^2 = C_F$

gluon (color) charge $\mathcal{Q}_g^2 = C_A$

$$\mathcal{Q}_1^2 + \mathcal{Q}_2^2 + 2\mathcal{Q}_1\cdot\mathcal{Q}_2 = \mathcal{Q}_3^2 \quad\Rightarrow\quad \mathcal{Q}_1\cdot\mathcal{Q}_2 = \frac{1}{2}\left(\mathcal{Q}_3^2 - \mathcal{Q}_2^2 - \mathcal{Q}_1^2\right)$$

Restriction of phase space

$$\left|(k|\mathcal{M}|p_1,p_2)\right|^2 = 4g^2\left[\mathcal{Q}_1^2\mathcal{P}_1 + \mathcal{Q}_2^2\mathcal{P}_2 + \mathcal{Q}_3^2\mathcal{P}_{12}\right]$$

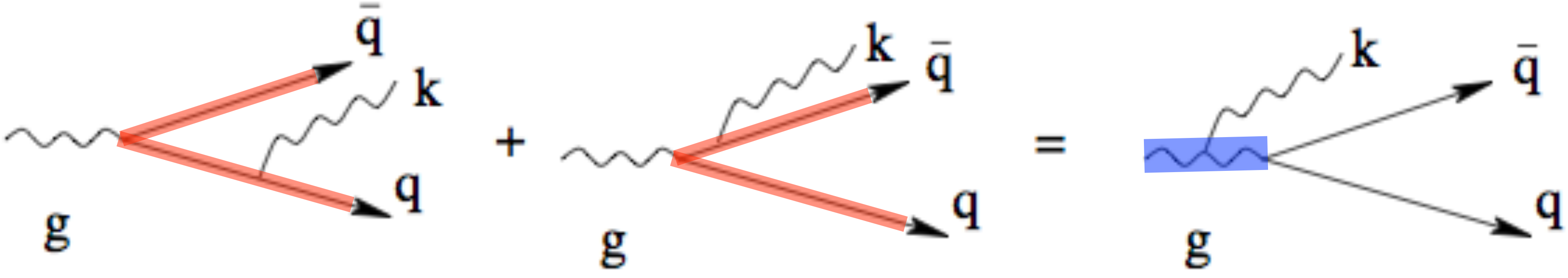
Coherent spectrum

$$\mathcal{P}_1 = \frac{1}{(\boldsymbol{k}-\omega n_1)^2}\left[1 - \frac{(\boldsymbol{k}-\omega n_1)\cdot(\boldsymbol{k}-\omega n_2)}{(\boldsymbol{k}-\omega n_2)^2}\right]$$

$$\int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}\,\mathcal{P}_1 = \frac{1}{\boldsymbol{k}_{\mathrm{rel}}^2}\,\Theta(\theta_{12}-\theta_k)$$

$$\mathrm{d}N \simeq \frac{\alpha_s N_c}{\pi}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\boldsymbol{k}_{\mathrm{rel}}^2}{\boldsymbol{k}_{\mathrm{rel}}^2} \quad\Rightarrow\quad \mathrm{d}N \simeq \frac{\alpha_s N_c}{\pi}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\boldsymbol{k}_{\mathrm{rel}}^2}{\boldsymbol{k}_{\mathrm{rel}}^2}\,\Theta(\theta_{12}-\theta_k)$$

Color magic



large-angle emissions are restored with the total charge!

Quark @ small angles

$$\omega \frac{dN_g}{d\omega d^2 k_\perp} \propto \frac{\alpha_s C_F}{k_\perp^2} + (q \to \bar{q})$$

$$\theta \ll \theta_{q\bar{q}} \; (k_\perp \ll \omega \theta_{q\bar{q}})$$

Gluon @ large angles

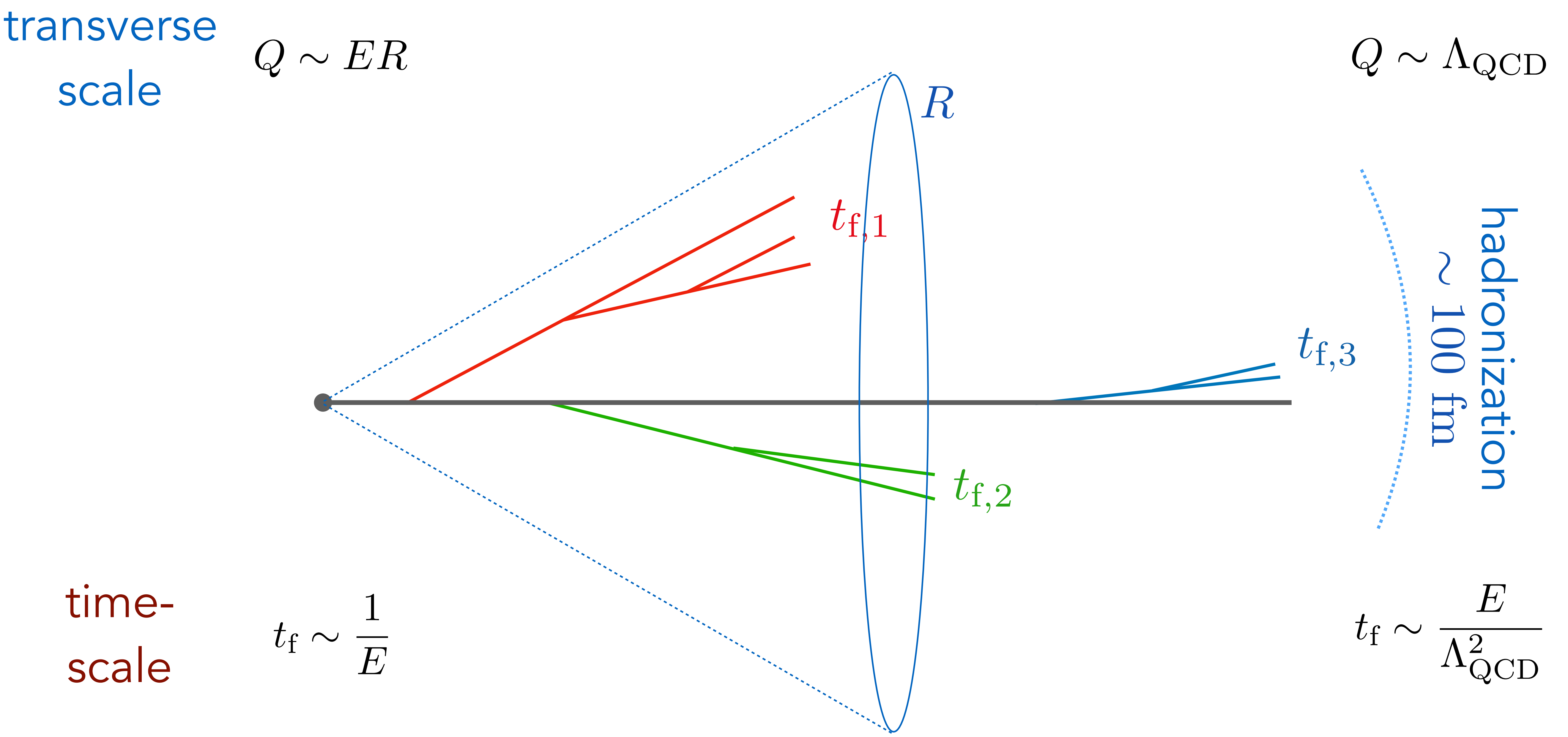
$$\omega \frac{dN_g}{d\omega d^2 k_\perp} \propto \frac{\alpha_s C_A}{k_\perp^2}$$

$$\theta \gg \theta_{q\bar{q}} \; (k_\perp \gg \omega \theta_{q\bar{q}})$$

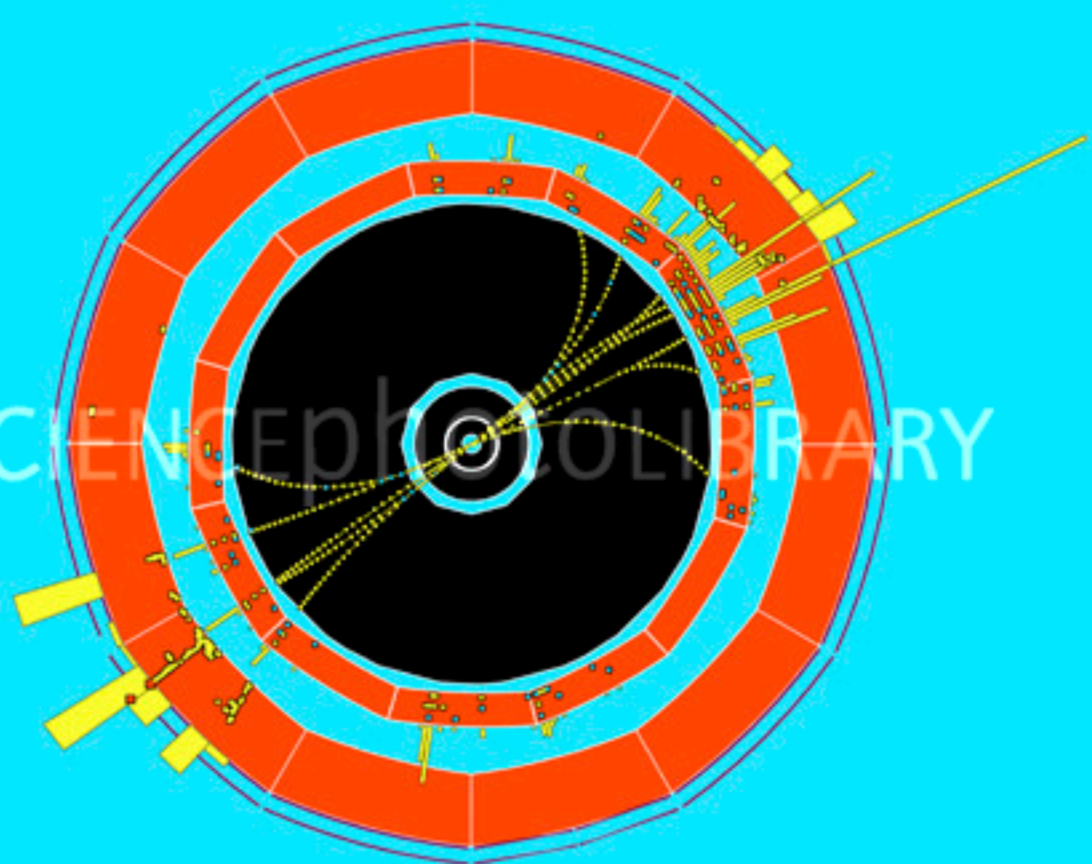
In MCs, this is implemented as condition of angular ordered emissions.

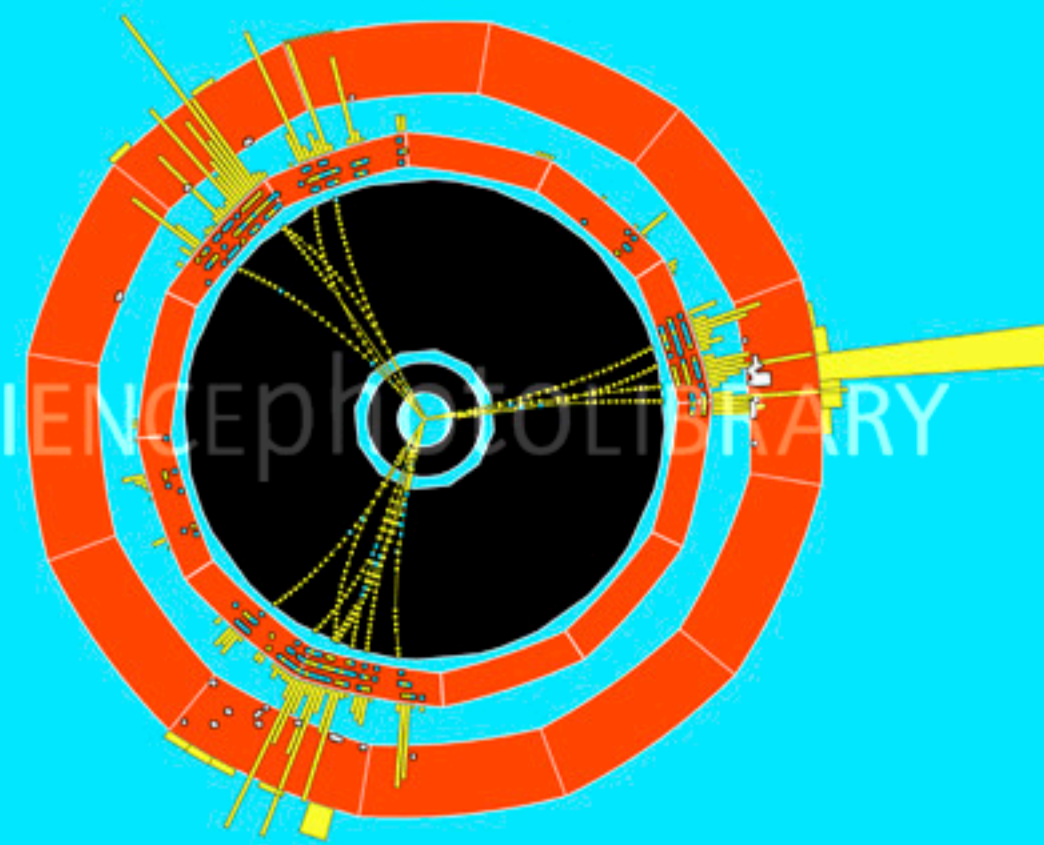
QCD PARTON SHOWER

Leading-logarithmic approximation: (strongly) ordered in energies, angles and, therefore, formation times



A extensive "technology" developed to make sense of images such as these:





Aim: link experimental measurements w/ theoretical expectations.

SUMMARY

- separation of short- and long-distance processes
  - perturbative calculations w/ quarks & gluons
- gluon emissions
  - bring about (IR) divergencies
  - soft & collinear radiation: we have to resum multiple emissions
    - ➡ perturbative evolution