Theory of the Deconfinement Transition and its Signatures
- Lecture 2 -

Chihiro Sasaki
Institute of Theoretical Physics
University of Wroclaw

Outline

Lecture 1: Brief introduction

Lecture 2: Interacting quarks and gluons

- Polyakov-loop model: deconfinement
- Nambu--Jona-Lasinio model: chiral SB

Lecture 3: Critical behaviors

- Phase transition and the Landau theory
- Fluctuations of conserved charges

1. Polyakov-loop model

Hidden global symmetry

QCD Lag. invariant under SU(3) gauge transf.

$$A_\mu(x) \to \Omega^\dagger(x) A_\mu(x) \Omega(x) + \frac{i}{g} \Omega^\dagger(x) \partial_\mu \Omega(x)\,, \;\; q(x) \to \Omega(x) q(x)$$

$$\Omega(x) \in SU(3) \quad \text{thus} \quad \Omega^\dagger(x)\Omega(x) = \mathbf{1}\,, \;\; \det \Omega(x) = 1$$

- Consider $\Omega(x)=\Omega_1$ as a subset:

$$\Omega_1 = \begin{pmatrix} e^{2\pi i/3} & 0 & 0 \\ 0 & e^{2\pi i/3} & 0 \\ 0 & 0 & e^{2\pi i/3} \end{pmatrix} = e^{2\pi i/3}\mathbf{1}\,, \;\; \det \Omega_1 = (e^{2\pi i/3})^3 = 1$$

- Under this transf.

$$A_\mu(x) \to e^{-2\pi i/3} A_\mu(x) e^{2\pi i/3} = A_\mu(x)\,, \;\; q(x) \to e^{2\pi i/3} q(x) \neq q(x)$$

invariant NOT invariant

Hidden global symmetry

- All such phases are

$$\Omega_1 = e^{2\pi i/3}\mathbf{1}\,,\ \ \Omega_2 = e^{-2\pi i/3}\mathbf{1}\,,\ \ \Omega_3 = \mathbf{1}$$

→When no quarks, the system is invariant under the above transfs.

→ Z(3) symmetry – discrete and global

- In SU(Nc), Z(Nc) symmetry transf. is given by

$$\Omega = e^{i\phi}\mathbf{1}\,,\ \ \phi = 2\pi j/N_c\ \ (j = 0, 1, \cdots, N_c - 1)$$

- NOTE: Quarks break this symmetry explicitly.

Z(3) invariants

- Polyakov loop

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{1/T} d\tau A_0(\vec{x},\tau)\right]$$

$$L(\vec{x}) \to \Omega^\dagger(\vec{x},1/T)L(\vec{x})\Omega(\vec{x},0)$$

$$\Omega(\vec{x},1/T) = e^{i\phi}\Omega(\vec{x},0)$$

- Under gauge transf.

$$\Phi \equiv (1/3)\mathrm{tr}\left[L(\vec{x})\right] \to e^{i2\pi n/3}\Phi$$,(n = 0,1,2)

→ Z(3) invariants are

$$\Phi^\dagger\Phi\ ,\ \ {\Phi^\dagger}^3 + \Phi^3$$

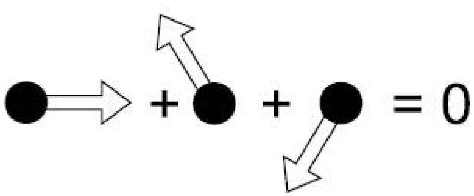
← building blocks of effective potential

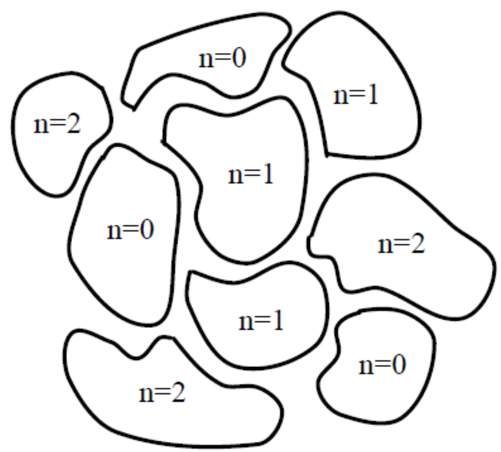
Confinement vs. Z(3)

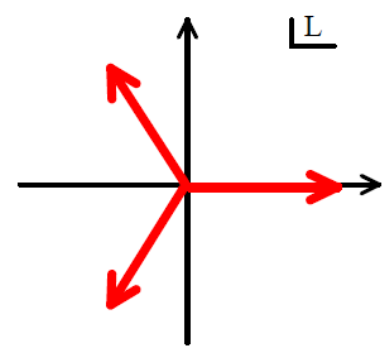
- Free energy of a static quark [McLerran and Svetitsky]

$$\langle \Phi(x) \rangle \sim e^{-F_q(x)/T} \begin{cases} = 0 & \text{confined, Z(3) unbroken} \\ \neq 0 & \text{deconfined, Z(3) broken} \end{cases}$$

- Confining vacuum:
  - Domains of Z(3) phases
  - Randomly distributed
  - → Average to zero







Deconfinement in YM

- ❑Effective potential [Pisarski (2000)]

$$\frac{\mathcal{U}(\Phi,\bar{\Phi};T)}{T^4} = -\frac{b_2}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

→Introducing T-dep. in b_2 → phase transition

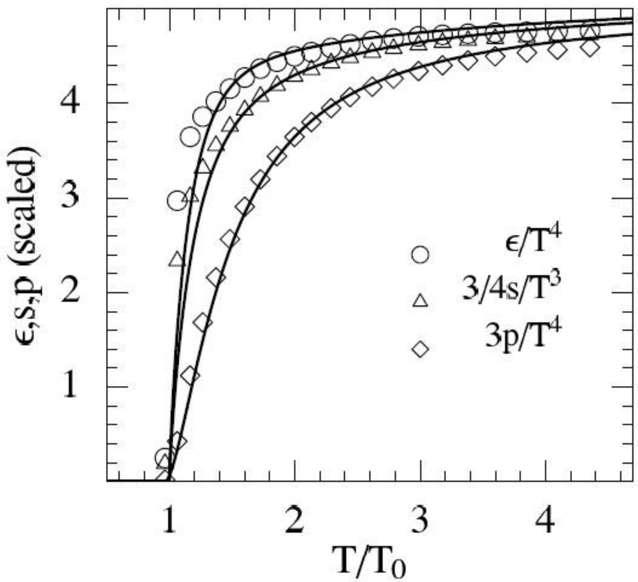
- ❑Fit to lattice YM-EoS [Ratti et al. (2006)]

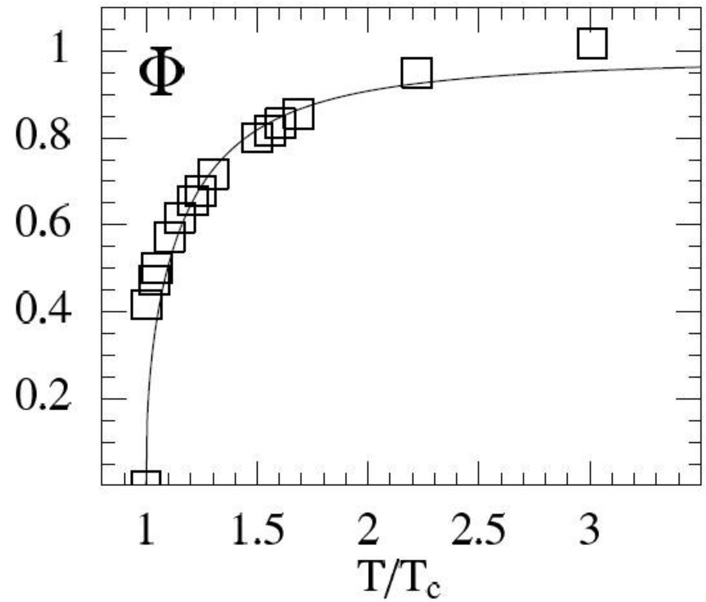
$$b_2(T) = a_0 + a_1\left(T_0/T\right) + a_2\left(T_0/T\right)^2 + a_3\left(T_0/T\right)^3$$

a_0	a_1	a_2	a_3	b_3	b_4
6.75	−1.95	2.625	−7.44	0.75	7.5

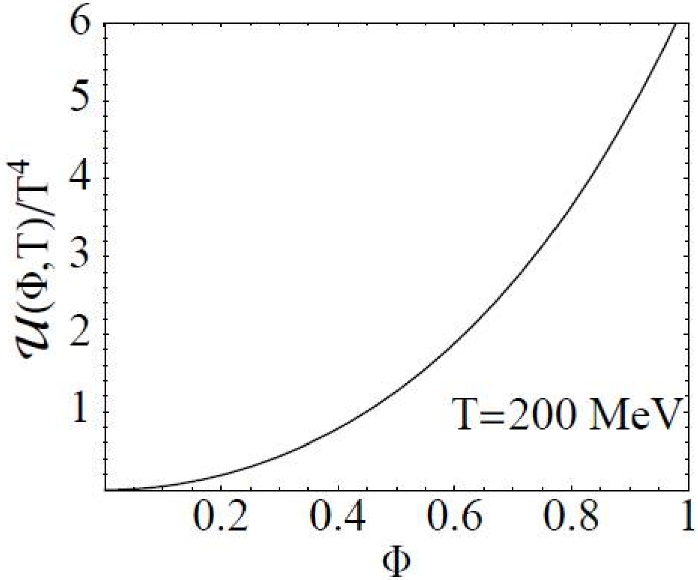
[Figures taken from Ratti et al. (2006)]

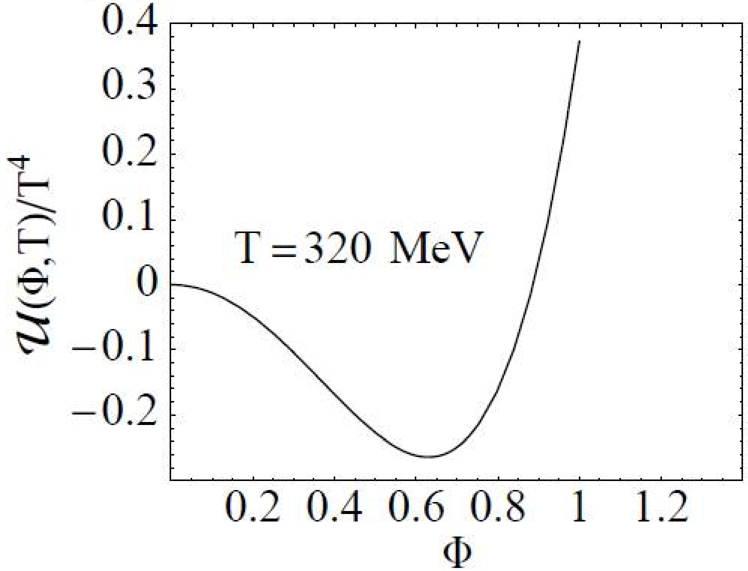
Deconfinement in YM





- minimizing U
- gap equation





[Technical details and applications in CS and Redlich (2012)]

A little bit more

- $b_2(T)$: where T-dep. comes from? → gluons

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$

$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0}\,, \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

$$\hat{L}_A(A^3_0, A^8_0) = \hat{L}_A(\phi_1, \phi_2)$$

- The entire potential $\Omega = \Omega_g + \Omega_{\rm Haar}$

$$\Omega_g = 2T\int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \sum_{n=1}^{8} C_n(\Phi,\bar{\Phi})\, e^{-n|\vec{p}|/T}\right),$$

1 const. parameter!

$$\Omega_{\rm Haar} = -a_0 T \ln\left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]$$

With quarks???

How good is the Z(3) symmetry in QCD?

- Good symmetry when mq → ∞
- Explicitly broken in QCD:
  - Heavy quarks: may be softly broken
  - Light quarks: badly broken
- (Naïve) implementation as in PNJL/PQM
  - Might be risky
  - Might be fine when looking at fluctuations?

Cf. Lo, Szymanski, Redlich and CS (2018)

2. Nambu—Jona-Lasinio model

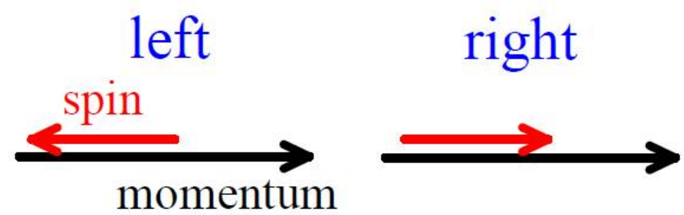
What emerges when mq → 0?

- Spin projection onto the momentum vector

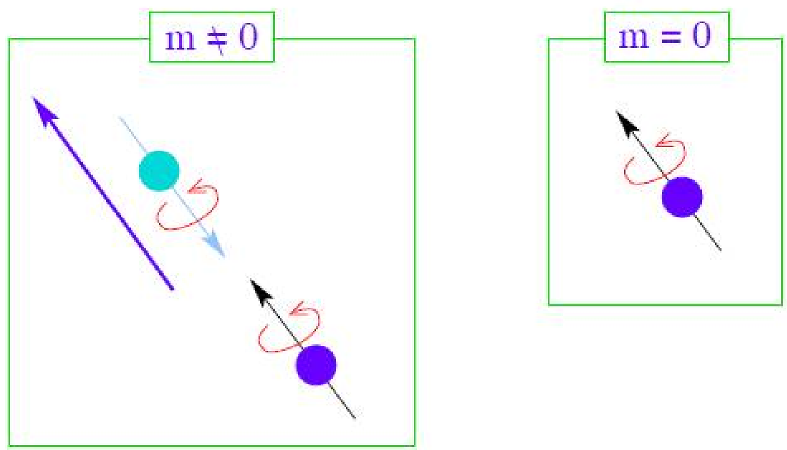
operator $\vec{\sigma}\cdot\vec{p}_{\text{unit}}/2 = \vec{s}\cdot\vec{p}_{\text{unit}}$ - *helicity*

ψ_L: helicity $-1/2$, ψ_R: helicity $1/2$ particles

$$q_L = \frac{1-\gamma_5}{2}q, \qquad q_R = \frac{1+\gamma_5}{2}q.$$



$$\mathcal{L} = \bar{q}_L i\gamma^\mu\partial_\mu q_L + \bar{q}_R i\gamma^\mu\partial_\mu q_R + m_q(\bar{q}_L q_R + \bar{q}_R q_L)$$



Mq=0 limit
→ No L-R mixing
→ Chiral symmetry

Spontaneous symmetry breaking

e.g. complex scalar theory

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

- Global U(1) symmetry and commutators

$$\phi \to \phi' = e^{i\theta}\phi \sim (1 + i\theta)\phi\,, \quad \phi^* \to \phi^{*\prime} = e^{-i\theta} \sim (1 - i\theta)\phi^*$$

$$[i\theta Q, \phi] = i\theta\phi = \delta\phi\,, \quad [i\theta Q, \phi^*] = -i\theta\phi^* = \delta\phi^*$$

- If $\mu^2 > 0$:

$$\langle 0|[Q,\phi]|0\rangle = \langle 0|\phi|0\rangle = 0 \qquad Q|0\rangle = \langle 0|Q = 0$$

- If $\mu^2 < 0$:

$$\langle 0|[Q,\phi]|0\rangle = \langle 0|\phi|0\rangle = v/\sqrt{2} \qquad Q|0\rangle \neq 0\,, \quad \langle 0|Q \neq 0$$

Spontaneous symmetry breaking

- Goldstone's theorem: *when a global continuous symmetry is spontaneously broken, a massless scalar particle emerges.*
- If a symmetry charge doesn't annihilate the vacuum, non-vanishing VEV emerges:

$$Q|0\rangle \neq 0\,, \qquad \langle 0|Q \neq 0$$
$$\langle 0|[Q,\phi]|0\rangle = \langle 0|\phi|0\rangle = v/\sqrt{2}$$

- QCD: pions (mpi = 140 MeV << mrho, mN) as approximate NG bosons (mq small but finite)

Consequences as LETs

- Partially conserved axial-current

$$\partial^\mu J^a_{A,\mu} = f_\pi m_\pi^2 \pi^a$$

$$J^a_{A,\mu} = i q_\mu f_\pi \pi^a = f_\pi \partial_\mu \pi^a$$

- Gell-Mann—Oakes—Renner relation

$$m_\pi^2 f_\pi^2 = -m_q \langle \bar{q} q \rangle$$

- Goldberger—Treiman relation

$$f_\pi g_{\pi NN} = 2 m_N g_A$$

Nambu—Jona-Lasinio model

Lagrangian $\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{G}{N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2\right]$

- N-component fermion

$$\psi = (\psi_1, \psi_2, \cdots, \psi_N)^T$$

- U(1)L x U(1)R chiral symmetry

$$\psi_R \to e^{i\theta_R}\psi_R\,, \quad \psi_L \to e^{i\theta_L}\psi_L$$

$$\mathcal{L} = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R + \frac{G}{N}(\bar{\psi}_L \psi_R)(\bar{\psi}_R \psi_L)$$

- Generating functional

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{i\int d^4x \mathcal{L}}$$

Effective potential

- ❑Bosonization $\sigma \sim \bar{\psi}\psi\,,\;\; \pi \sim \bar{\psi}\gamma_5\psi$
- ❑Large N approximation

→ leading-order eff. Potential

$$
\begin{aligned}
V(\sigma,\pi) &= \frac{1}{4G}(\sigma^2+\pi^2) - \int \frac{d^4k}{i(2\pi)^4}\ln\det\left[\not{k} - \sqrt{\sigma^2+\pi^2}\right] \\
&= \frac{1}{4G}(\sigma^2+\pi^2) - 2\int \frac{d^4k}{i(2\pi)^4}\ln\left[\sigma^2+\pi^2-k^2\right]\,.
\end{aligned}
$$

→ k-integral needs to be regulated --- cutoff Λ

Self-consistent equation

- Expanding it for a large Λ;

$$V(\sigma,\pi) - V(0,0) = \left(\frac{1}{4G} - \frac{\Lambda^2}{8\pi^2}\right)\tilde{\sigma}^2 \equiv \left(\frac{1}{4G} - \frac{1}{4G_{\rm cr}}\right)\tilde{\sigma}^2$$

with $\tilde{\sigma}^2 \equiv \sigma^2 + \pi^2$ and $G_{\rm cr} = \dfrac{2\pi^2}{\Lambda^2}$ Critical coupling

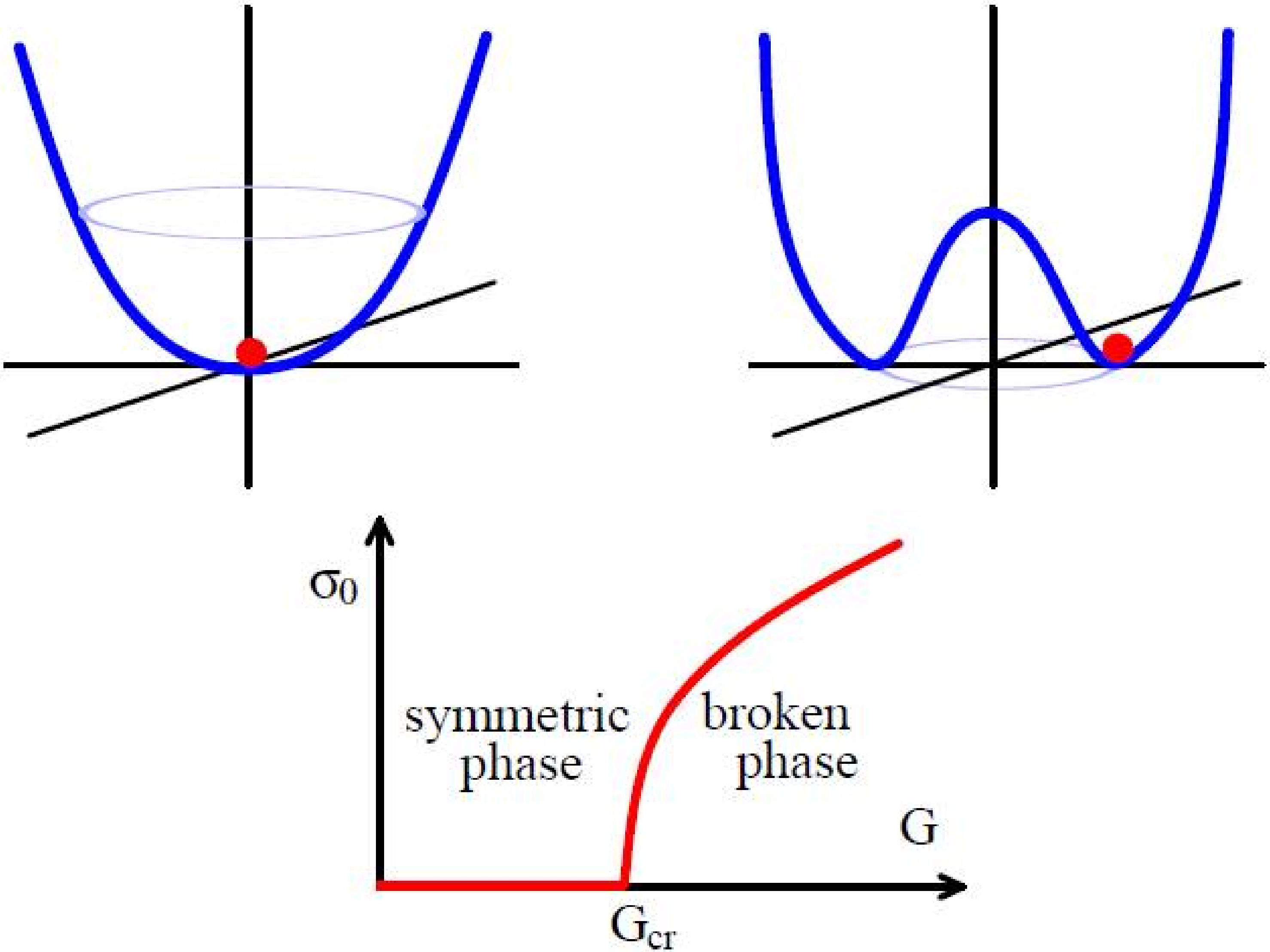
- Stationary condition

$$0 = \frac{\partial V}{\partial \sigma}\Big|_{\sigma=\sigma_0} = \sigma_0\left[\frac{1}{2G} - \frac{1}{2G_{\rm cr}} + \frac{\sigma_0^2}{4\pi^2}\ln\left(1 + \frac{\Lambda^2}{\sigma_0^2}\right)\right]$$

- If G < Gcr, trivial solution $\sigma_0 = 0$
- If G > Gcr, 2 solutions, $\sigma_0 = 0$ and $\sigma_0 \neq 0$

→ $V(0) > V(\sigma_0 \neq 0)$

Phase diagram



[e.g. Meisinger-Ogilvie, Fukushima, Ratti et al., CS et al., Schaefer et al.]

Putting Polyakov-loops

Lagrangian based on chiral & Z(3)

$$\mathcal{L} = \bar{\psi}(i\not{D} - m + \gamma_0\mu)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \mathcal{U}(\Phi, \bar{\Phi}; T)$$

- ❑Covariant derivative

$$D_\mu = \partial_\mu - iA_\mu\,, \quad A_\mu = \delta_{\mu 0}A^0$$

- ❑Thermodynamics potential – mimicking conf.

$$\Omega(T,\mu) = \mathcal{U} + \frac{(m-M)^2}{2G} - 12\int\frac{d^3p}{(2\pi)^3}E_p - 4T\int\frac{d^3p}{(2\pi)^3}\left\{\ln g^{(+)} + \ln g^{(-)}\right\}$$

with

$$g^{(+)} = 1 + 3\,\Phi\,e^{-(E_p-\mu)/T} + 3\,\bar{\Phi}\,e^{-2(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}$$
$$g^{(-)} = 1 + 3\,\bar{\Phi}\,e^{-(E_p+\mu)/T} + 3\,\Phi\,e^{-2(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}$$

QCD Phase transition

- Modifications of particle properties due to a phase transition – critical behaviors
- How to quantify them?

→ QCD vs. models: the same symmetry

- Universality hypothesis: critical behaviors are model-independent