Initial stages of heavy ion collisions and small x

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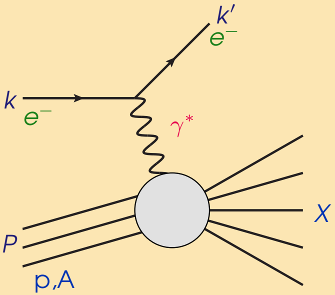
Lecture plan

1. DIS and BK equation: high energy evolution, TRF view of gluon saturation
2. Wilson lines: how the gluon field appears at small x.
3. IMF view of gluon saturation, initial stage of heavy ion collision

1 Deep Inelastic Scattering in the dipole picture

DIS kinematics, high energy=small x

How to probe what a proton or nucleus is made of



$$
\begin{aligned}
s &= (k+P)^2 \\
q &= k - k' \quad q^2 \equiv -Q^2 \\
W^2 &= (P+q)^2 \\
x &= \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2\nu m_N} = \frac{Q^2}{W^2 + Q^2 - m_N^2}
\end{aligned}
$$

$(\nu = P \cdot q / m_N)$

$$\left(y = \frac{2P\cdot q}{2P\cdot k} = \frac{W^2 + Q^2 - m_N^2}{s m_N^2} \right)$$

High energy limit is $x \to 0$

- This is when $W^2 \to \infty$; $\nu \to \infty$; i.e. the virtual photon-target c.m.s. energy is high.
- Now Q^2 is "fixed". In DGLAP the limit is x fixed, Q^2 large (large transverse momentum)

I want to convince you that the γ^* is the theorist's favorite hadron!

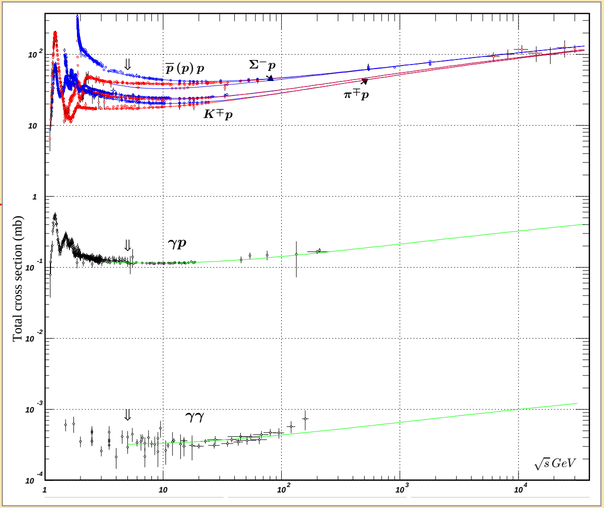
Cross sections vs. energy

γ scatters just like *p*

— apart from extra $\frac{1}{137}$

The same should be true for γ^*

PDG cross sections $\Longrightarrow$



Kinematical variables in TRF

TRF = target rest frame

Light cone coordinates
$x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)$

(Note $\mathbf{x}_\perp$ is 2d transverse)

$$P^\mu = (\overset{0}{m}, \overset{\perp}{\mathbf{0}}, \overset{z}{0}) \Longrightarrow (\overset{+}{m/\sqrt{2}}, \overset{-}{m/\sqrt{2}}, \overset{\perp}{\mathbf{0}})$$

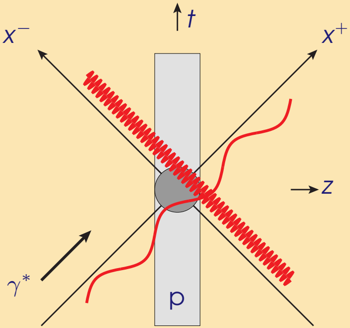
$$q^\mu = (\overset{0}{W^2/(2m)}, \overset{\perp}{\mathbf{0}}, \overset{z}{\sqrt{(W^2/(2m))^2 + Q^2}}) \Longrightarrow (\overset{+}{q^+}, \overset{-}{-Q^2/(2q^+)}, \overset{\perp}{\mathbf{0}})$$

High energy: $q^+ \approx W^2/(\sqrt{2}m)$ **big**

Look at γ^* wavefunction $e^{-i(q^+x^- + q^-x^+)}$

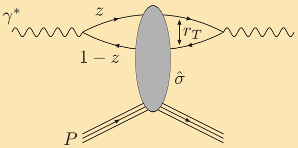
- Very accurate resolution in x^-
- No resolution in x^+
  Scattering instantaneous in x^+ compared to natural timescale of γ^*

In particular γ^* cannot change into a hadronic final state **inside** proton; it has to fluctuate into hadrons before.



DIS in dipole picture

Simplest hadronic state in the interacting γ^* state: quark-antiquark dipole.



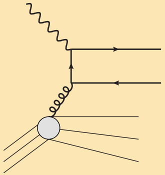
$$\sigma_{T,L}^{\gamma^* p} = \int d^2\mathbf{r}_\perp \, dz \left| \psi^{\gamma^* \to q\bar{q}}(\mathbf{r}_\perp, z)_{T,L} \right|^2 2\mathcal{N}$$

High energy: we assume (lifetime/timescale) factorization between

- $\left| \psi^{\gamma^* \to q\bar{q}}(\mathbf{r}_\perp, z)_{T,L} \right|^2$: probability for photon to fluctuate into $\bar{q}q$ dipole: QED process
- $\mathcal{N}$ = imaginary part of the forward elastic γ^*-p/A scattering amplitude, i.e. half the total cross section; optical theorem

Same process in the IMF would look like this

- Formally higher order (NLO DIS)
- Dominates at small x because $xg(x, Q^2)$ is large
- Does not describe valence quarks



Virtual photon wavefunction $\psi^{\gamma^* \to q\bar{q}}$

The concept makes sense in the framework of
Light **C**one **P**erturbation **T**heory: (No time to go very far here)

Outline of LCPT calculation

- Idea: know free particle Fock states: $|\gamma^*\rangle_0, \quad |q\bar{q}\rangle_0, \quad |q\bar{q}g\rangle_0$ etc.
- **Interacting** states are superpositions of these:

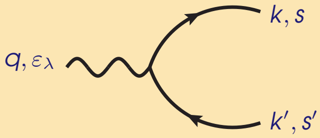
$$|\gamma^*\rangle = (1 + \dots)|\gamma^*\rangle_0 + \psi^{\gamma^* \to q\bar{q}} \otimes |q\bar{q}\rangle_0 + \psi^{\gamma^* \to q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

- QM perturbation theory: ground state $|0\rangle$ wavefunction correction is

$$\sum_n \frac{\langle n| \hat{V} |0\rangle}{E_n - E_0} |n\rangle$$

  - Here $1/\Delta E$ is $\sim$ the lifetime of the quantum fluctuation from 0 to n
- In LCPT, "energy" is k^-
- Matrix elements $\langle n| \hat{V} |0\rangle$ are vertices in Feynman rules

Calculating $\psi^{\gamma^*\to q\bar{q}}$



Need two things to calculate $\psi^{\gamma^*\to q\bar{q}}$

- Matrix element

$$e\bar{u}_s(k)\not\varepsilon_\lambda v_{s'}(k') \quad ; \quad s, s' = \pm\frac{1}{2}; \quad \lambda = 0 = L, \quad \lambda = \pm 1 = T$$

- Energy denominator $(q^- - k^- - k'^-)^{-1}$

$$= -\left(\frac{Q^2}{2q^+} + \frac{\mathbf{k}_\perp^2 + m^2}{2zq^+} + \frac{\mathbf{k}_\perp^2 + m^2}{2(1-z)q^+}\right) = \frac{-2q^+z(1-z)}{\underbrace{Q^2z(1-z) + m^2}_{\equiv \varepsilon^2} + \mathbf{k}_\perp^2}$$

Fourier-transform $\mathbf{k}_\perp \to \mathbf{r}_\perp$, sum over spins; result is

$$\left|\psi_T^{\gamma^*\to q\bar{q}}\right|^2 = \frac{\alpha_{\text{e.m.}}}{2\pi^2} N_c e_f \left(\left[z^2 + (1-z)^2\right] K_1^2(\varepsilon r) + m_f^2 K_0^2(\varepsilon r)\right)$$

$$\left|\psi_L^{\gamma^*\to q\bar{q}}\right|^2 = \frac{\alpha_{\text{e.m.}}}{2\pi^2} N_c e_f 4Q^2 z^2 (1-z)^2 K_0^2(\varepsilon r)$$

DIS dipole frame: summary

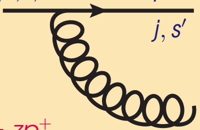
- Picture DIS as γ^* scattering on target
- At high energy (in TRF) γ^* fluctuates into $q\bar{q}$

$$\sigma^{\gamma^* p}_{T,L} = \int \mathrm{d}^2\mathbf{r}_\perp \, \mathrm{d}z \left|\psi^{\gamma^*\to q\bar{q}}(r,z)_{T,L}\right|^2 2\mathcal{N} \left|\psi^{\gamma^*\to q\bar{q}}(r,z)_{T,L}\right|^2 \sim \exp\left\{\sqrt{z(1-z)}Qr\right\}$$

- Typical dipole size: $r \sim 1/Q$
- Used optical theorem: $2\mathcal{N}$ is total cross section
  - can also take $|\mathcal{N}|^2$:connection to elastic scattering (diffractive DIS)
- We are assuming that fixed-size dipoles are the basis that diagonalizes the imaginary part of the T-matrix
  - This makes sense in an eikonal approximation for the scattering
  - In general: high energy/eikonal approximation: particles fly through target at fixed $\mathbf{x}_\perp$; Does not imply zero momentum transfer! Rather energy is so high, that momentum transfer does not change $\mathbf{x}_\perp$ during interaction

2 Balitsky-Kovchegov equation

What happens if one radiates a gluon?



Light cone wavefunction

$$\psi^{q\to qg}(z, \mathbf{k}_\perp) = \frac{1}{\frac{\mathbf{p}_\perp^2}{2p^+} - \frac{\mathbf{k}_\perp^2}{2k^+} - \frac{\mathbf{p}'^{\,2}_\perp}{2p'^+}} \bar{u}_{s'}(p')(-g)t^a_{ji}\not{\varepsilon}(k)u_s(p)$$

Matrix elements e.g. from Pauli hep-ph/0103106

This is simple in the **soft** limit $z \to 0$:

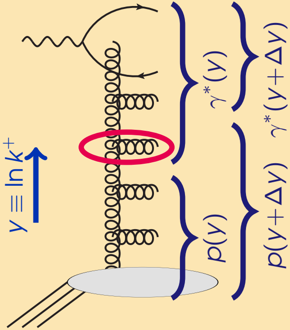
$$\psi^{q\to qg}(k^+, \mathbf{k}_\perp) = \frac{-2zp^+}{\mathbf{k}_\perp^2}\frac{-2gt^a_{ji}}{z}\boldsymbol{\varepsilon}_\perp \cdot \mathbf{k}_\perp \delta_{s,s'} \qquad \text{“}\mathrm{d}P_{q\to qg}\text{''} = \left|\psi^{q\to qg}\right|^2 \frac{\mathrm{d}k^+\,\mathrm{d}^2\mathbf{k}_\perp}{2k^+(2\pi)^3} \sim \frac{\mathrm{d}z}{z}\frac{\mathrm{d}^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2}$$

Typical gauge theory logarithmic divergences in emission probability:

$$\textbf{soft}\quad \frac{\mathrm{d}z}{z} \quad - \quad \textbf{collinear}\quad \frac{\mathrm{d}^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2} \qquad \left(\sum_{\lambda=\pm1}\varepsilon_i\varepsilon_j^* = \delta_{ij}\right)$$

Soft gluons and large logs, idea of RGE

- Emitted gluons have z between 1 and $x \sim 1/W^2$: each gluon contributes $\sim \alpha_s \ln 1/x$
- For x small $\alpha_s \ln 1/x \sim 1 \implies$ all n gluon emissions contribute same $\implies$ resum
- Done by Renormalization Group Equation



Is the gluon at y a part of γ^* or of p?

You have to decide! Physical cross section is the same.

gluons up to y are part of proton

$$\sigma^{\gamma^* p} = \underbrace{\left|\psi^{\gamma^* \to q\bar{q}}\right|^2_y \otimes 2\mathcal{N}_y^{q\bar{q}p} + \left|\psi^{\gamma^* \to q\bar{q}g}\right|^2_y \otimes 2\mathcal{N}_y^{q\bar{q}gp} + \dots}$$

$$= \left|\psi^{\gamma^* \to q\bar{q}}\right|^2_{y+\Delta y} \otimes 2\mathcal{N}_{y+\Delta y}^{q\bar{q}p} + \left|\psi^{\gamma^* \to q\bar{q}g}\right|^2_{y+\Delta y} \otimes 2\mathcal{N}_{y+\Delta y}^{q\bar{q}gp} + \dots$$

gluons up to $y+\Delta y$ are part of proton

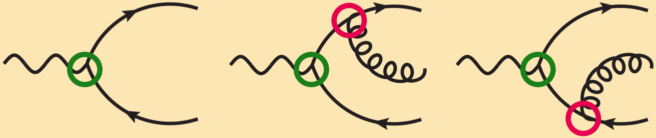
Can calculate $\left|\psi^{\gamma^* \to q\bar{q}}\right|^2_y$'s $\implies$ get differential equation for unknown $\mathcal{N}_y$

Gluon emission from coordinate space dipole

Let's put this idea into practice. We will

- Calculate $\psi^{\gamma^* \to q\bar{q}g}(z)$
- Take soft gluon limit $z \to 0$
- Reabsorb the gluon to become a part of the target
- Get evolution equation for $q\bar{q}$ cross section

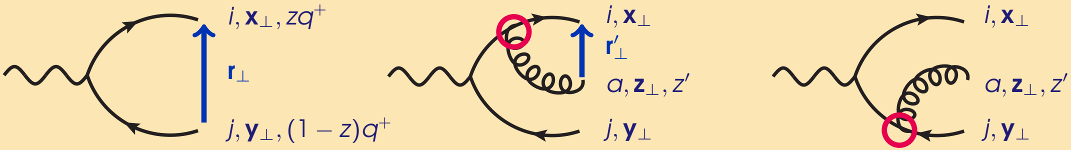
We need:



We can do this with $\psi^{\gamma^* \to q\bar{q}}$ we already know and and coordinate space

$$\psi^{q\to qg}(k^+, \mathbf{r}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \psi^{q\to qg}(k^+, \mathbf{k}_\perp) = -2ip^+ \frac{2gt^a_{ji}}{2\pi} \frac{\varepsilon_\perp \cdot \mathbf{r}_\perp}{\mathbf{r}_\perp{}^2} \delta_{s,s'}$$

Gluon emission from coordinate space dipole



$$|\gamma^*\rangle_{\text{int}} = |\gamma^*\rangle + \int_{z,\mathbf{r}_\perp} C(\mathbf{r}_\perp)\psi^{\gamma^*\to q\bar{q}}(z,\mathbf{r}_\perp)\,|q_i(\mathbf{x}_\perp,z)\bar{q}_i(\mathbf{y}_\perp,1-z)\rangle$$

$$+\int_{z,\mathbf{r}_\perp,\mathbf{r}'_\perp}\psi^{\gamma^*\to q\bar{q}}(z,\mathbf{r}_\perp)\int\frac{\mathrm{d}z'}{4\pi z'}\frac{-i2g}{2\pi}t^a_{ji}\left[\frac{(\mathbf{x}_\perp-\mathbf{z}_\perp)\cdot\boldsymbol{\varepsilon}_\perp}{(\mathbf{x}_\perp-\mathbf{z}_\perp)^2}-\frac{(\mathbf{y}_\perp-\mathbf{z}_\perp)\cdot\boldsymbol{\varepsilon}_\perp}{(\mathbf{y}_\perp-\mathbf{z}_\perp)^2}\right]|q_i(\mathbf{x}_\perp)\bar{q}_j(\mathbf{y}_\perp)g_a(\mathbf{z}_\perp)\rangle\,,$$

Adjust coefficient of $q\bar{q}$-state to keep wavefunction normalized:

$$N_c\,|C(\mathbf{r}_\perp)|^2 = N_c - \frac{(2g)^2}{(2\pi)^2}\frac{1}{4\pi}t^a_{ij}t^a_{ji}\int\frac{\mathrm{d}z'}{z'}\int\mathrm{d}^2\mathbf{r}'_\perp\sum_{\lambda=\pm1}\left|\frac{(\mathbf{x}_\perp-\mathbf{z}_\perp)\cdot\boldsymbol{\varepsilon}_{\perp\lambda}}{(\mathbf{x}_\perp-\mathbf{z}_\perp)^2}-\frac{(\mathbf{y}_\perp-\mathbf{z}_\perp)\cdot\boldsymbol{\varepsilon}_{\perp\lambda}}{(\mathbf{y}_\perp-\mathbf{z}_\perp)^2}\right|^2$$

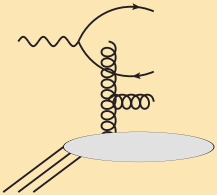
$$= N_c - \frac{\alpha_s}{\pi^2}\frac{N_c{}^2-1}{2}\Delta y\int\mathrm{d}^2\mathbf{r}'_\perp\frac{\mathbf{r}_\perp{}^2}{\mathbf{r}'_\perp{}^2(\mathbf{r}_\perp-\mathbf{r}'_\perp)^2}$$

$$\sum_{\lambda=\pm1}\varepsilon_i^{(\lambda)}\varepsilon_j^{(\lambda)*}=\delta_{ij}$$

Crucial step: move the gluon to the target

Scattering amplitude is $\mathcal{N}(\mathbf{r}_\perp) = \int \mathrm{d}^2\mathbf{b}_\perp \mathcal{N}(\mathbf{b}_\perp, \mathbf{r}_\perp)$.
We want equality between scattering amplitudes with gluon in different place:

$$\mathcal{N}_{q\bar{q}}^{y+\Delta y} = \mathcal{N}_{q\bar{q}}^{y} + \frac{\alpha_s}{\pi^2}\frac{N_c{}^2-1}{2N_c}\int_y^{y+\Delta y} \mathrm{d}\ln 1/z' \int \mathrm{d}^2\mathbf{r}'_\perp \frac{\mathbf{r}_\perp{}^2}{\mathbf{r}'_\perp{}^2(\mathbf{r}_\perp-\mathbf{r}'_\perp)^2}\left[\mathcal{N}_{q\bar{q}g}^{\ln 1/z'} - \mathcal{N}_{q\bar{q}}^{\ln 1/z'}\right]$$



Dipole scattering on new target $\mathcal{N}_{q\bar{q}}^{y+\Delta y}$ is

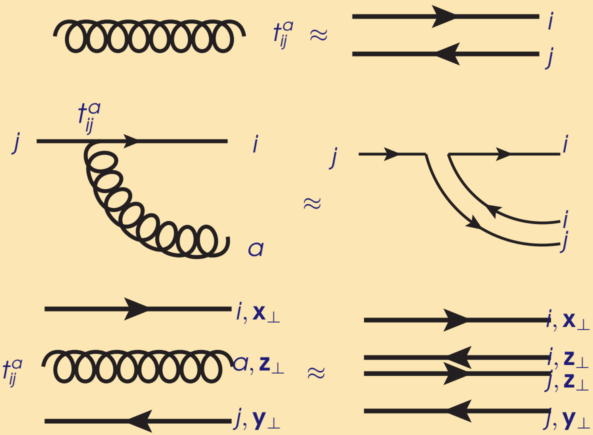
- Dipole scattering off original target $\mathcal{N}_{q\bar{q}}^{y}$
- Dipole emits a gluon into rapidity interval $[y, y+\Delta y]$, which scatters off target
- Normalization of original dipole is corrected (There are now less dipoles in γ^*)

Almost there

We are looking for an equation for $\mathcal{N}_{q\bar{q}}$: but enocuntered new quantity $\mathcal{N}_{q\bar{q}g}$, which needs to be related to $\mathcal{N}_{q\bar{q}}$. Will do this in the large N_c approximation

Gluon at large N_c

- At large N_c $\Longrightarrow$ gluon $= q\bar{q}$ pair (not dipole!)
- $N_c{}^2 - 1$ gluon colors $\approx N_c{}^2$ quark-antiquark pair colors.
- Had $|q(\mathbf{x}_\perp)\bar{q}(\mathbf{y}_\perp)g(\mathbf{z}_\perp)\rangle$
- Approximate by $|q(\mathbf{x}_\perp)\bar{q}(\mathbf{z}_\perp)q(\mathbf{z}_\perp)\bar{q}(\mathbf{y}_\perp)\rangle$



Now, instead of $\mathcal{N}_{q\bar{q}g}$, we need $\mathcal{N}_{q\bar{q}q\bar{q}}$: amplitude for simultaneous scattering of two dipoles.

Two dipole scattering amplitude

- $\mathcal{N}$ is really **scattering probability**;
- $S = 1 - \mathcal{N}$ is probability **not to scatter**

For two dipoles:

- No scattering: neither dipole scatters $\Longrightarrow S_{q\bar{q}q\bar{q}} = S_{q\bar{q}} S_{q\bar{q}}$
- Scattering probability $\mathcal{N}_{q\bar{q}q\bar{q}} = 1 - S_{q\bar{q}q\bar{q}} = 1 - (1 - \mathcal{N}_{q\bar{q}})(1 - \mathcal{N}_{q\bar{q}})$

Thus we end up with the approximation:

$$\mathcal{N}^{q\bar{q}g}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp) \approx \mathcal{N}^{q\bar{q}}(\mathbf{x}_\perp, \mathbf{z}_\perp) + \mathcal{N}^{q\bar{q}}(\mathbf{z}_\perp, \mathbf{y}_\perp) - \mathcal{N}^{q\bar{q}}(\mathbf{x}_\perp, \mathbf{z}_\perp)\mathcal{N}^{q\bar{q}}(\mathbf{z}_\perp, \mathbf{y}_\perp)$$

and our equation is

$$\mathcal{N}^{q\bar{q}}_{y+\Delta y} = \mathcal{N}^{q\bar{q}}_{y} + \frac{\alpha_s}{\pi^2}\frac{N_c{}^2 - 1}{2N_c}\int_y^{y+\Delta y} \mathrm{d}\ln 1/z' \int \mathrm{d}^2\mathbf{z}_\perp \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2}{(\mathbf{x}_\perp - \mathbf{z}_\perp)^2(\mathbf{z}_\perp - \mathbf{y}_\perp)^2}$$
$$\times \left[\mathcal{N}^{q\bar{q}}_{\ln 1/z'}(\mathbf{x}_\perp, \mathbf{z}_\perp) + \mathcal{N}^{q\bar{q}}_{\ln 1/z'}(\mathbf{z}_\perp, \mathbf{y}_\perp) - \mathcal{N}^{q\bar{q}}_{\ln 1/z'}(\mathbf{x}_\perp, \mathbf{z}_\perp)\mathcal{N}^{q\bar{q}}_{\ln 1/z'}(\mathbf{z}_\perp, \mathbf{y}_\perp) - \mathcal{N}^{q\bar{q}}_{\ln 1/z'}(\mathbf{x}_\perp, \mathbf{y}_\perp)\right]$$

Differentially for infinitesimal Δy, and with large N_c

$$\partial_y \mathcal{N}(\mathbf{r}_\perp) = \frac{\alpha_s N_c}{2\pi^2}\int \mathrm{d}^2\mathbf{r}'_\perp \frac{\mathbf{r}_\perp{}^2}{\mathbf{r}'_\perp{}^2(\mathbf{r}'_\perp - \mathbf{r}_\perp)^2}\left[\mathcal{N}(\mathbf{r}'_\perp) + \mathcal{N}(\mathbf{r}_\perp - \mathbf{r}'_\perp) - \mathcal{N}(\mathbf{r}'_\perp)\mathcal{N}(\mathbf{r}_\perp - \mathbf{r}'_\perp) - \mathcal{N}(\mathbf{r}_\perp)\right]$$

Summary

Balitsky-Kovchegov equation (∼1995)

$$\partial_y \mathcal{N}(\mathbf{r}_\perp) = \frac{\alpha_s N_c}{2\pi^2} \int d^2\mathbf{r}'_\perp \frac{{\mathbf{r}_\perp}^2}{{\mathbf{r}'_\perp}^2 (\mathbf{r}'_\perp - \mathbf{r}_\perp)^2} \left[\mathcal{N}(\mathbf{r}'_\perp) + \mathcal{N}(\mathbf{r}_\perp - \mathbf{r}'_\perp) - \mathcal{N}(\mathbf{r}'_\perp)\mathcal{N}(\mathbf{r}_\perp - \mathbf{r}'_\perp) - \mathcal{N}(\mathbf{r}_\perp)\right]$$

This is the basic tool of modern small-x physics.

- Given initial condition $\mathcal{N}(\mathbf{r}_\perp)$ at $y = y_0$ the equation predicts the scattering amplitude at larger y = smaller x = higher $\sqrt{s}$.
- Drop nonlinear term: BFKL equation
- Divergences at $\mathbf{r}'_\perp \to 0$ and $\mathbf{r}'_\perp \to \mathbf{r}_\perp$ regulated because $\mathcal{N}(0) = 0$ due to color neutrality.
- Enforces black disk limit (unitraity) $\mathcal{N} < 1$
- For practical work coupling α_s should depend on distance: some combination of $\mathbf{r}_\perp, \mathbf{r}'_\perp, \mathbf{r}_\perp - \mathbf{r}'_\perp$

What the solution of BK looks like

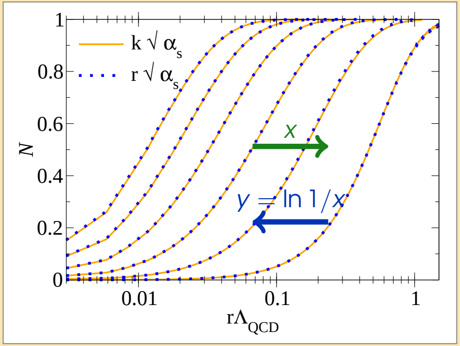
The equation can be solved numerically

- Small dipoles $r \lesssim 1/Q_s$ scatter very little
  At $r = 0$ color neutral system, should not scatter by the strong interaction!
- Large dipoles $r \gtrsim 1/Q_s$ scatter with probability almost one, but not more. **Saturation**

Remember, for the DIS F_2, F_L convolute this with the (known) γ^* wavefunction.

$$\sigma^{\gamma^* p}_{T,L} = \int \mathrm{d}^2\mathbf{b}_\perp\, \mathrm{d}^2\mathbf{r}_\perp\, \mathrm{d}z \left|\psi^{\gamma^*\to q\bar{q}}(r,z)_{T,L}\right|^2 2\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp, x)$$

Fits HERA data ($x < 0.01$ Q^2 moderate) extremely well
(*b*-dependence modeled with varying degrees of sophistication)



(Actually cheating, this plot is a solution of JIMWLK, which generalizes BK)

3 Eikonal propagation in target color field

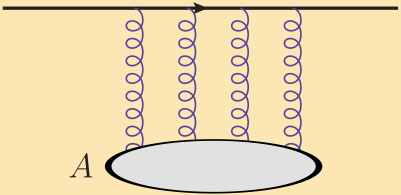
What is the target made of?

- So far we have not specified anything about the degrees of freedom in the target.
- We will srgue that at high energy the target consists dominantly of gluons
  - We know that at small *x* the gluon distribution is larger than the quark one.
  - BK equation builds up the target by adding gluons to it.

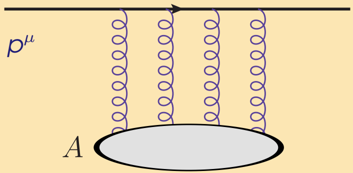
Color Glass Condensate (CGC)

We assume that there are so many gluons in the target, that it can be described by a classical gluon field. This is the heart of the CGC effective theory.

Many gluons = large color field A_μ
Have to sum all diagrams with *n* gluons lines
— but we can assume the gluons are a classical field



What is the target made of?



Quark propagating in classical color field: Dirac equation!

$$(i\partial\!\!\!/ - gA\!\!\!/)\psi(x) = 0$$

(Note: $A\!\!\!/ = A^\mu_a \gamma_\mu t^a$ is $N_c \times N_c$-matrix)

Want to dig out the dominant contribution: **eikonal** approximation

- Gluon is spin 1: it couples to a vector: $\sim p^\mu A_\mu$
- For high energy particle the only momentum available is p^μ
- p^μ has one large component: $p^+ \implies p^\mu A_\mu \sim p^+ A^- \implies$ only need A^-

Ansatz for DE: $\psi(x) = V(x)e^{-ip\cdot x}u(p)$, plug into equation

$$\implies \partial_+ V(x^+, x^-, \mathbf{x}_\perp) = -igA^-(x^+, x^-, \mathbf{x}_\perp)\mathbf{V}(x^+, x^-, \mathbf{x}_\perp)$$

$N_c \times N_c$-matrix!

This is solved by path-ordered exponential

$$V(x^+, x^-, \mathbf{x}_\perp) = \mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x}_\perp)\right\}$$

Eikonal propagation

- Now we know how a high energy quark propagates in a classical field.
- Thus we know the scattering S-matrix element for many-quark states

E.g. for a quark the time evolution operator is

$$\hat{S}\,(x^+ = -\infty \to x^+ = \infty)\,|q(i,\mathbf{x}_\perp)\rangle = \left[\mathbb{P}\exp\left\{-ig\int_{-\infty}^{\infty} \mathrm{d}y^+ A^-(y^+,x^-,\mathbf{x}_\perp)\right\}\right]_{ji} |q(j,\mathbf{x}_\perp)\rangle$$

$\Longrightarrow$ quark becomes linear superposition of quarks at same $\mathbf{x}_\perp$, different color states.

- In scattering problem integrate $x^+ \in [-\infty, \infty]$
- In the high energy limit quark wavefunction oscillates like $e^{ip^+x^-}$ with large p^+
  $\Longrightarrow$ x^--dependence negligible compared to this $\Longrightarrow$ approximate $x^- = 0$

Scattering is described by 2-dimensional field of SU(N_c)-matrices

$$V(\mathbf{x}_\perp) \equiv \mathbb{P}\exp\left\{-ig\int_{-\infty}^{\infty} \mathrm{d}x^+ A^-(x^+, x^- = 0, \mathbf{x}_\perp)\right\}$$

This is the **Wilson line**

Dipole amplitude and Wilson lines

Incoming dipole (color neutral, average over colors!) becomes

$$\hat{S}\,\frac{\delta_{ij}}{\sqrt{N_c}}\,|q_i(\mathbf{x}_\perp)\bar{q}_j(\mathbf{y}_\perp)\rangle = \frac{\delta_{ij}}{\sqrt{N_c}}\,V_{i'i}(\mathbf{x}_\perp)V^*_{j'j}(\mathbf{y}_\perp)\,|q'_i(\mathbf{x}_\perp)\bar{q}_{j'}(\mathbf{y}_\perp)\rangle = \frac{1}{\sqrt{N_c}}\left[V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\right]_{i'j'}|q_{i'}(\mathbf{x}_\perp)\bar{q}_{j'}(\mathbf{y}_\perp)\rangle$$

The total cross section is related to the imaginary part of the **forward elastic scattering amplitude**; i.e. project out dipole in outgoing state

$$\frac{\delta_{k\ell}}{\sqrt{N_c}}\,\langle q_k(\mathbf{x}_\perp)\bar{q}_\ell(\mathbf{y}_\perp)|\,\hat{S}\,\frac{\delta_{ij}}{\sqrt{N_c}}\,|q_i(\mathbf{x}_\perp)\bar{q}_j(\mathbf{y}_\perp)\rangle = \frac{1}{N_c}\,\mathrm{Tr}\left[V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\right]$$

Dipole amplitude in the CGC

Relate $\mathcal{N}$ in BK and DIS to a **microscopical description of the target**:

$$\mathcal{N}_{q\bar{q}} = 1 - \frac{1}{N_c}\,\mathrm{Tr}\,V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)$$

Note $\quad S_{fi} = \langle f|\,\hat{S}\,|f\rangle = 1 + iT_{fi} \quad \sigma_{\mathrm{tot}} = 2\mathrm{Im}T_{ii} \quad \mathcal{N} \equiv \mathrm{Im}T_{ii} \quad S_{ii} = \delta_{ii} - \mathcal{N} + \mathrm{imag}$

More complicated operators

- The dipole amplitude is a target expectation value of a two-point function

$$\mathcal{N}_{q\bar{q}} = 1 - \left\langle \hat{D} \right\rangle = \left\langle 1 - \frac{1}{N_c} \mathrm{Tr}\, V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \right\rangle_{\text{target}}$$

- For this we derived the BK equation using a **mean field** approximation $\left\langle \hat{D}\hat{D} \right\rangle \approx \left\langle \hat{D} \right\rangle \left\langle \hat{D} \right\rangle$
- Similarly define other correlators, such as $\left\langle \hat{D}\hat{D} \right\rangle$ or the quadrupole

$$Q = \left\langle \frac{1}{N_c} \mathrm{Tr}\, V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) V(\mathbf{u}_\perp) V^\dagger(\mathbf{v}_\perp) \right\rangle_{\text{target}},$$

  and the corresponding evolution equations.
- Without the mean field approx. these operators couple to each other (e.g. $\partial_y \left\langle \hat{D} \right\rangle \sim \left\langle \hat{D}\hat{D} \right\rangle$) the **Balitsky hierarchy** of evolution equations
- The hierarchy can be generalized into an evolution equation for the **probability distribution of Wilson lines** — the JIMWLK equation

From BK to JIMWLK

JIMWLK equation

Gives rapidity-dependence of probability distribution of Wilson lines

$$\partial_y W_y[V(\mathbf{x}_\perp)] = \mathcal{H} W_y[V(\mathbf{x}_\perp)]$$

$$\mathcal{H} \equiv \frac{1}{2} \int_{\mathbf{x}_\perp \mathbf{y}_\perp \mathbf{z}_\perp} \frac{\delta}{\delta \mathcal{A}_c^-(\mathbf{y}_\perp)} \mathbf{e}_\perp{}^{ba}(\mathbf{x}_\perp, \mathbf{z}_\perp) \cdot \mathbf{e}_\perp{}^{ca}(\mathbf{y}_\perp, \mathbf{z}_\perp) \frac{\delta}{\delta \mathcal{A}_b^-(\mathbf{x}_\perp)},$$

$$\mathbf{e}_\perp{}^{ba}(\mathbf{x}_\perp, \mathbf{z}_\perp) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}_\perp - \mathbf{z}_\perp}{(\mathbf{x}_\perp - \mathbf{z}_\perp)^2} \left(1 - V^\dagger(\mathbf{x}_\perp) V(\mathbf{z}_\perp)\right)^{ba}$$

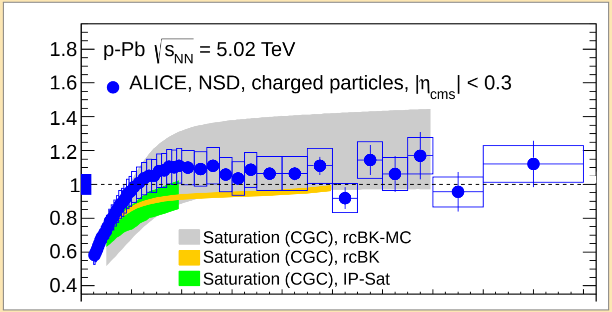
You can derive this in a very similar way as we did for BK.

- Assume there is a y-dependent probability distribution $W_y[U(\mathbf{x}_\perp)]$
- Consider collection of n Wilson lines propagating through target
- Emit one extra soft gluon and absorb small-z divergence into redefinition of probability distribution: $W_y[V(\mathbf{x}_\perp)] \to W_{y+\Delta y}[V(\mathbf{x}_\perp)]$

4 Particle production in pA

Nuclear modification factor R_{pA}

Comparison of ALICE data on particle production in pA and pp to some theory predictions



There are two ways to calculate this in the CGC

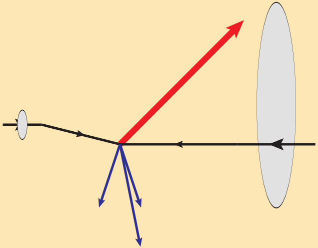
k_T-factorization Good at midrapidity/symmetric situation with strong color fields in **both** colliding objects. This we will come to a bit later.

Hybrid formalism One colliding object described as dilute collection of partons $\Longrightarrow$ good at forward rapidity. Let us first understand this.

Dilute-dense scattering

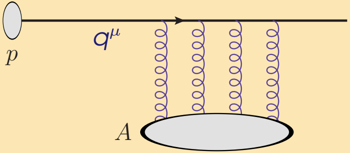
Look at forward rapidity pA

- The produced particle has large p^+.
- Momentum conservation:
  large p^+ needs to come from large x parton in proton
- At large x proton = dilute collection of valence quarks
  $\Longrightarrow$ quark scattering on dense target



In: quark with momentum $q^+, \mathbf{q}_\perp$, color i

$$|\text{in}\rangle = \int \mathrm{d}^2\mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} |q(i, \mathbf{x}_\perp)\rangle$$

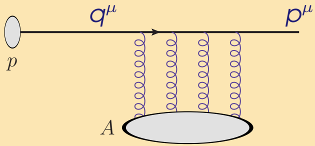


After interaction with the target

$$\hat{S}\,|\text{in}\rangle = \int \mathrm{d}^2\mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} V_{ji}(\mathbf{x}_\perp)\,|q(j, \mathbf{x}_\perp)\rangle$$

Scattering amplitude

$$|\text{in}\rangle = \int \mathrm{d}^2\mathbf{x}_\perp e^{-i\mathbf{q}_\perp\cdot\mathbf{x}_\perp} V_{ji}(\mathbf{x}_\perp)\,|q(j,\mathbf{x}_\perp)\rangle$$



Amplitude: quarks with momentum $\mathbf{p}_\perp$ in the final state

(Should also subtract the 1 in $S = 1 + iT$, but this is a δ-function)

$$\mathcal{M}_{i,\mathbf{q}_\perp\to k,\mathbf{p}_\perp} = \langle q(k,\mathbf{p}_\perp)|\,\text{in}\rangle = \int_{\mathbf{x}_\perp,\bar{\mathbf{x}}_\perp} e^{-i(\mathbf{q}_\perp\cdot\mathbf{x}_\perp - \mathbf{p}_\perp\cdot\bar{\mathbf{x}}_\perp)} V_{ji}(\mathbf{x}_\perp)\overbrace{\langle q_k(\bar{\mathbf{x}}_\perp)|\; q(\mathbf{x}_\perp)_j\rangle}^{\delta^2(\bar{\mathbf{x}}_\perp-\mathbf{x}_\perp)\delta_{kj}}$$

Incoming quark is collinear $\mathbf{q}_\perp = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}_\perp} = \frac{1}{N_c}\frac{1}{(2\pi)^2}\sum_{i,k}|\mathcal{M}_{i,\mathbf{q}_\perp\to k,\mathbf{p}_\perp}|^2 = \frac{1}{N_c}\frac{1}{(2\pi)^2}\int_{\mathbf{x}_\perp,\mathbf{y}_\perp} e^{-i\mathbf{p}_\perp\cdot(\mathbf{x}_\perp-\mathbf{y}_\perp)}\,\mathrm{Tr}\,V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)$$

There are $xq(x,\mu^2)$ incoming quarks in the proton per unit rapidity.

Hybrid formula for quark production

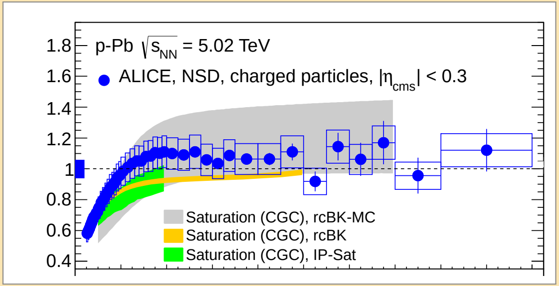
$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}_\perp\,\mathrm{d}y} = \frac{1}{(2\pi)^2}xq(x,\mu^2)\frac{1}{N_c}\int \mathrm{d}^2\mathbf{x}_\perp\,\mathrm{d}^2\mathbf{y}_\perp\, e^{-i\mathbf{p}_\perp\cdot(\mathbf{x}_\perp-\mathbf{y}_\perp)}\,\mathrm{Tr}\,V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)$$

Back to R_{pA}

$$\frac{d\sigma}{d^2\mathbf{q}_\perp\, dy} = \frac{1}{(2\pi)^2} xq(x,\mu^2) \times \frac{1}{N_c}\int_{\mathbf{x}_\perp \mathbf{y}_\perp} e^{-i\mathbf{q}_\perp\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)}\,\mathrm{Tr}\, V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)$$

Now all we need is a parametrization, for protons **and** nuclei of

$$\mathrm{Tr}\, V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)$$



- Fit to HERA data $\Longrightarrow$ proton dipole amplitude
  - using BK equation (remember: BK gives x-dependence, need to fit initial condition)
  - or some other model of the dipole cross section (IPsat)
- Generalize to nuclei: somehow incorporate Woods-Saxon $T_A(b)$
  - rcBK-MC and rcBK are different implementations of this
- The HERA data is very precise and (LO) theory fits it well: the "theory errors" in the above plot are all from this proton $\Longrightarrow$ nucleus generalization.

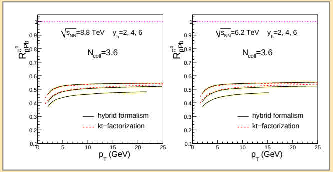
From protons to nuclei

One typical initial condition for BK: GBW Golec-Biernat, Wusthoff:

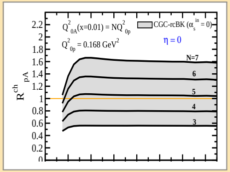
$$\mathcal{N}(\mathbf{b}_\perp, \mathbf{r}_\perp) = \theta(R_p - b)\left(1 - \exp\left\{-\frac{{\mathbf{r}_\perp}^2}{4{Q_{s,p}}^2}\right\}\right), \qquad \text{and for nucleus?}$$

1. Just fit $Q_{s,A}$ separately to some nuclear data
2. Assume saturation scale $Q_s^2 \sim T_A(\mathbf{b}_\perp)$ or $A^{1/3}$ — but with what coefficient?
3. MC Glauber, N_N overlapping nucleons and $(Q_{s,A})^2 = N_N\,(Q_{s,A})^2$ — Fine, but what is nucleon area for calculating N_N? Same as in DIS? Same as in Glauber? (These are different!)

One has to be careful (I'm being nasty showing these celebrated plots)



Oops!



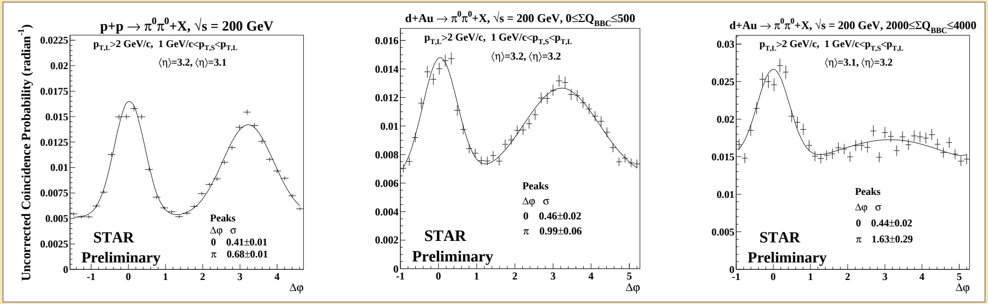
And the prediction was?

Differences mostly from nuclear geometry, not in the CGC theory

Another interesting observable forward dihadron correlations in dAu

Two particle correlation vs. $\Delta\varphi$:

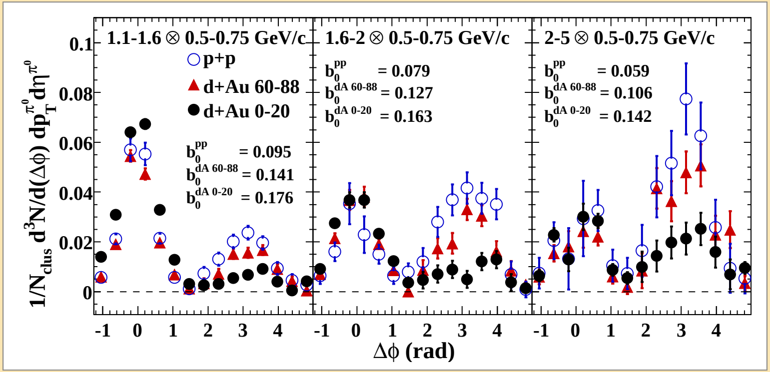
pp peripheral *d*Au central *d*Au



STAR, [arXiv:1102.0931]

Another interesting observable forward dihadron correlations in dAu

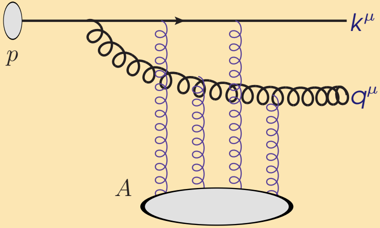
Two particle correlation vs. $\Delta\varphi$:



PHENIX, [arXiv:1105.5112], PRL

Calculating 2-particle correlation in forward pA

C. Marquet Nucl. Phys. A **796** (2007) 41; (arXiv:0708.0231 (hep-ph)).



- Quark from p (large x) from pdf, radiate gluon $\Longrightarrow$ light cone wave function
- Eikonally through target $\Longrightarrow$ Wilson lines
  - Fundamental reps $V(\mathbf{x}_\perp)$ for quark
  - Adjoint reps $U(\mathbf{x}_\perp)$ for gluon $\Longrightarrow$ Fierz
- Need target exp. values of Wilson line operators — from JIMWLK (or an approximation thereof)

$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3\mathbf{q}\,\mathrm{d}^3\mathbf{k}} \propto \int_{\mathbf{x}_\perp,\bar{\mathbf{x}}_\perp,\mathbf{y}_\perp,\bar{\mathbf{y}}_\perp} [\cdots] \left\langle \hat{Q}(\mathbf{y}_\perp,\bar{\mathbf{y}}_\perp,\bar{\mathbf{x}}_\perp,\mathbf{x}_\perp)\,\hat{D}(\mathbf{x}_\perp,\bar{\mathbf{x}}_\perp) - \hat{D}(\mathbf{y}_\perp,\mathbf{x}_\perp)\hat{D}(\mathbf{x}_\perp,\bar{\mathbf{z}}_\perp) + \dots \right\rangle_{\text{target}}$$

($\mathbf{z}_\perp = z\mathbf{x}_\perp + (1-z)\mathbf{y}_\perp$, $\bar{\mathbf{z}}_\perp = z\bar{\mathbf{x}}_\perp + (1-z)\bar{\mathbf{y}}_\perp$; $[\cdots]$= calculable LC wavefunction squared)

"Dipole" and "Quadrupole" operators

$$\hat{D}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) \qquad \hat{Q}(\mathbf{x}_\perp,\mathbf{y}_\perp,\mathbf{u}_\perp,\mathbf{v}_\perp) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)V(\mathbf{u}_\perp)V^\dagger(\mathbf{v}_\perp)$$

5 Gluon saturation and the CGC

Classical field and equation of motion

- We were describing the high energy nucleus as a classical field: A^- $\Longrightarrow$ Wilson line
- What does this imply for the partonic content of the nucleus?
- The physical picture of "gluons as partons" requires two things:
  - Infinite momentum frame: nucleus moving fast
    change direction: nucleus moves now in $+z$-direction with large p^+, large A^+
  - Light cone gauge: have to gauge transform to $A^+ = 0$
- CGC EFT based on separation of scales:
  - small x: classical field
  - large x: classical color charge

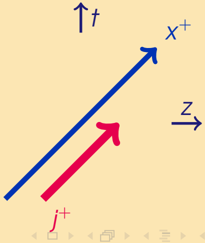
Classical $\equiv$ equation of motion

$$[D_\mu, F^{\mu\nu}] = J^\mu$$

What remains is

$$\nabla_\perp^2 A^+ = J^+$$

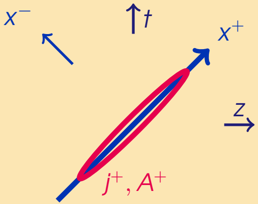
This is nice, **color** current in $+$-direction causes big A^+-field.



Spacetime structure of the field

The current lives on the light cone.

1. Naive explanation: Nucleus is Lorentz-contracted to $\Delta z \sim 2R_A m_A/\sqrt{s}$
2. Real explanation: Current represents large x degrees of freedom
   - Current: large p^+; field: small p^+
   - Current more localised in x^- than field.



Extreme approximation:

The current is independent of LC time x^+: **glass!**
Argument as above:

1. Naively: time is dilated for the nucleus
2. Any probe will have larger k^- than color current $\Longrightarrow$ probe oscillates faster in x^+ and sees current as static (in LC time x^+).

$$j^+(x^-, \mathbf{x}_\perp) \approx \delta(x^-)\rho(\mathbf{x}_\perp)$$

$$A^+(x^-, \mathbf{x}_\perp) \approx \delta(x^-)\frac{1}{\boldsymbol{\nabla}_\perp^2}\rho(\mathbf{x}_\perp)$$

Classical field and equation of motion

Now let us gauge transform.

$$A^+ \Rightarrow V^\dagger(\mathbf{x}_\perp, x^-)A^+V(\mathbf{x}_\perp, x^-) - \frac{i}{g}V^\dagger(\mathbf{x}_\perp, x^-)\partial_- V(\mathbf{x}_\perp, x^-) = 0$$

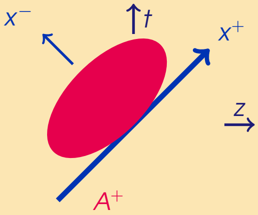
$$A^- \Rightarrow -\frac{i}{g}V^\dagger(\mathbf{x}_\perp, x^-)\partial_+ V(\mathbf{x}_\perp, x^-) = 0, \text{ still}$$

$$A^i \Rightarrow \frac{i}{g}V^\dagger(\mathbf{x}_\perp, x^-)\partial_i V(\mathbf{x}_\perp, x^-)$$

transverse pure gauge

This is solved by familiar Wilson line

$$V(\mathbf{x}_\perp, x^-) = \mathbb{P}\exp\left[-ig\int^{x^-} \mathrm{d}y^- A^+\right]$$



Now

$$A^i \sim \theta(x^-)$$

— delocalized in x^-, just like small k^+ physical gluons should be.

Weizsäcker-Williams gluon distribution

In LC quantization LC-gauge gluon field $A^i_a(\mathbf{k}_\perp)$ $\Longrightarrow$ number distribution of gluons:

$$\frac{dN}{d^2\mathbf{k}_\perp\, dy} \sim \left\langle A^i_a(\mathbf{k}_\perp) A^i_a(-\mathbf{k}_\perp) \right\rangle \qquad A^i = \frac{i}{g} V^\dagger(\mathbf{x}_\perp, x^-)\partial_i V(\mathbf{x}_\perp, x^-)$$

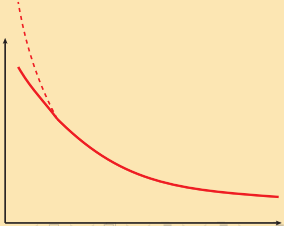
- DIS cross section, BK $\Longrightarrow$ Wilson line $\Longrightarrow$ gluon distribution
- One can express this **Weizsäcker-Williams** gluon distribution as:

$$\frac{dN}{d^2\mathbf{k}_\perp\, dy} = \varphi^{\mathrm{WW}}(\mathbf{k}_\perp) = \frac{C_{\mathrm{F}}}{2\pi^3}\frac{1}{\alpha_{\mathrm{s}}}\int d^2\mathbf{b}_\perp \int d^2\mathbf{r}_\perp \frac{e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}}{{\mathbf{r}_\perp}^2}\widetilde{\mathcal{N}}(\mathbf{b}_\perp, \mathbf{r}_\perp)$$

($\widetilde{\mathcal{N}}$ is the adjoint representation Wilson line correlator)

- Gluon saturation in $\varphi^{\mathrm{WW}}(\mathbf{k}_\perp)$ at $\mathbf{k}_\perp \lesssim Q_{\mathrm{s}}$
- $\varphi^{\mathrm{WW}}(\mathbf{k}_\perp) \sim 1/\alpha_{\mathrm{s}}$ $\Longrightarrow$ "**condensate**" of gluons

Now we have a **C**olor **G**lass **C**ondensate.



McLerran-Venugopalan model

L. D. McLerran and R. Venugopalan, Phys. Rev. D **49** (1994) 2233 & Phys. Rev. D **49** (1994) 3352

- Useful: explicit 1-parameter model for $\rho(\mathbf{x}_\perp, x^-)$ $\Longrightarrow$ easy to calculate
- Formally large A limit: independent valence-like color charges, CLT $\Longrightarrow$ Gaussian

MV model for charge density $\rho(\mathbf{x}_\perp, x^-)$

- Stochastic, Gaussian random field
- Local in x^- (infact very general) and $\mathbf{x}_\perp$ (can be generalized)

$$\left\langle \rho^a(\mathbf{x}_\perp, x^-)\rho^b(\mathbf{y}_\perp, y^-)\right\rangle = g^2\delta^{ab}\mu^2(x^-)\delta(x^- - y^-)\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

Calculate e.g. dipole cross section $\Longrightarrow$ identify saturation scale

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\right\rangle = \exp\left\{-\frac{g^4 C_F}{8\pi}\left[\int_{-\infty}^{\infty} dx^-\mu^2(x^-)\right] r^2 \ln\frac{1}{r\Lambda}\right\}$$

$$\Longrightarrow \quad Q_s^2 \sim \frac{g^4 C_F}{4\pi}\left[\int_{-\infty}^{\infty} dx^-\mu^2(x^-)\right]$$

6 Heavy ion collisions and the glasma initial state

Gluon fields in AA collision

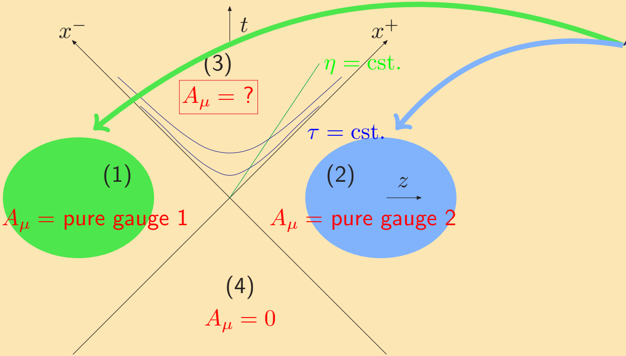
Now two colliding nuclei $\Longrightarrow$ two color currents

$$J^\mu = \delta^{\mu+}\rho_{(1)}(\mathbf{x}_\perp)\delta(x^-) + \delta^{\mu-}\rho_{(2)}(\mathbf{x}_\perp)\delta(x^+)$$

Classical **Y**ang-**M**ills

2 pure gauges from Wilson lines of 2 nuclei

$$A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$



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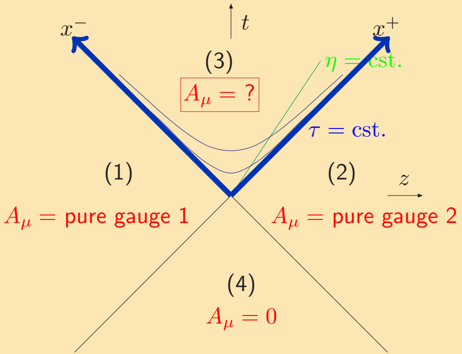


2 pure gauges from Wilson lines of 2 nuclei

$$A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

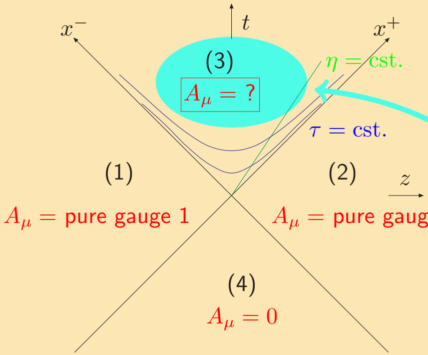
$$A^\eta\big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

Gluon fields in AA collision

Now two colliding nuclei $\Longrightarrow$ two color currents

$$J^\mu = \delta^{\mu+}\rho_{(1)}(\mathbf{x}_\perp)\delta(x^-) + \delta^{\mu-}\rho_{(2)}(\mathbf{x}_\perp)\delta(x^+)$$

Classical **Y**ang-**M**ills



2 pure gauges from Wilson lines of 2 nuclei

$$A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

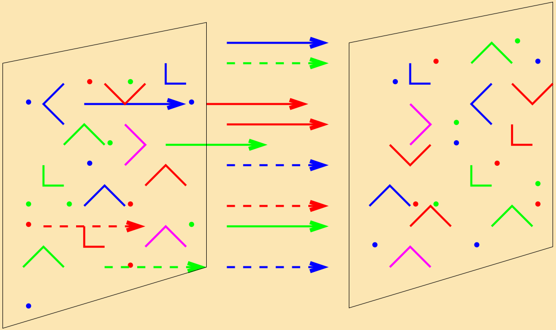
$$A^\eta\Big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

For $\boldsymbol{\tau > 0}$ solve numerically

This is the **glasma** field

$\Longrightarrow$ Then average over $\boldsymbol{\rho}$.

Result: glasma field



- $\tau = 0^+$: longitudinal E and B field,
- Depend on transverse coordinate with correlation length $1/Q_s$

$\Longrightarrow$ gluon correlations

Gauss law and Bianchi: (here $i = 1 \ldots 3$)

$$[D_i, E^i] = 0$$
$$[D_i, B^i] = 0$$

Separate nonabelian parts:

$$\partial_i E^i = ig[A^i, E^i]$$
$$\partial_i B^i = ig[A^i, B^i]$$

Effective E and M charge densities, Arising of interaction between pure gauge potential of one nucleus and the E/M field of the other

Deriving the initial condition

Let's work in Fock-Schwinger/temporal gauge $A_\tau = \left(x^+A^- + x^-A^-\right)/\tau = 0$
$\Longrightarrow$ consistent with LC gauge solutions for both nuclei.

$$\text{Ansatz: } A_i = \overbrace{A_i^{(1)}\theta(-x^+)\theta(x^-) + A_i^{(2)}\theta(x^+)\theta(-x^-)}^{\text{known}} + A_i^{(3)}\theta(x^+)\theta(x^-)$$

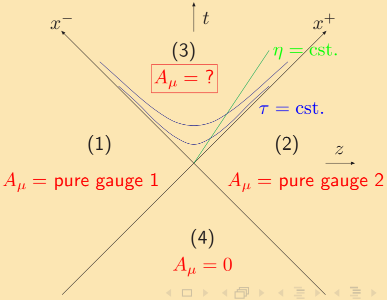
$$A^\pm = \pm\theta(x^+)\theta(x^-)x^\pm A^\eta$$

Insert into $[D_\mu, F^{\mu\nu}]$ and match coefficients of

- $\delta(x^+)\delta(x^-) \Longrightarrow A_i^{(3)}|_{\tau=0} = A_i^{(1)} + A_i^{(2)}$
- $\delta(x^+)\theta(x^-) \Longrightarrow A^\eta|_{\tau=0} = \frac{ig}{2}\left[A_i^{(1)}, A_i^{(2)}\right]$

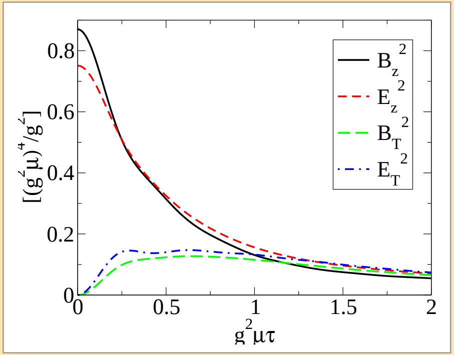
▼

Initial condition for region (3)



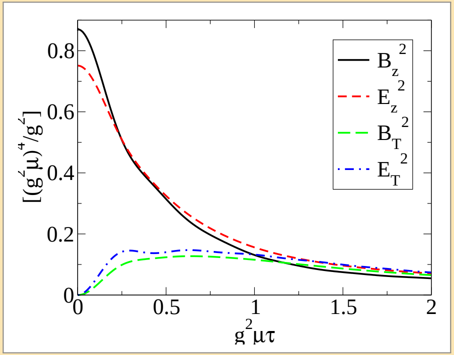
Boost invariant time evolution

Time evolution for field components



Boost invariant time evolution

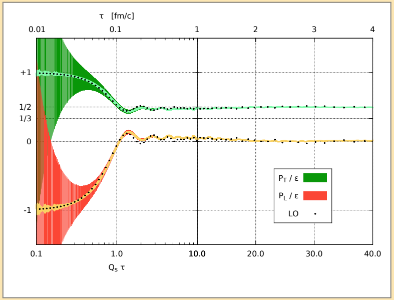
Time evolution for field components



Energy-momentum tensor $T_{\mu\nu}$:

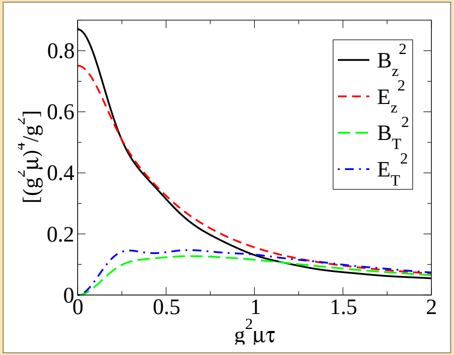
$$\varepsilon = \frac{1}{2}\left[E_x^2 + E_y^2 + E_z^2 + B_x^2 + B_y^2 + B_z^2\right]$$

$$p_x = \frac{1}{2}\left[-E_x^2 + E_y^2 + E_z^2 - B_x^2 + B_y^2 + B_z^2\right] \ldots$$



Boost invariant time evolution

Time evolution for field components



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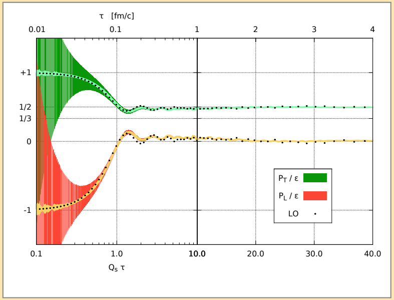


Boost invariance $\Longrightarrow$ $T_{\tau\tau} \sim \frac{1}{\tau}$; $T_{zz} \approx 0$

Gluon spectrum in the glasma

- CYM equations can be solved numerically on the lattice.
- Decompose solution in Fourier $\mathbf{k}_\perp$-modes: gluon spectrum

Q_s is only dominant scale

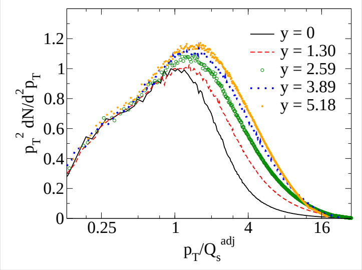
Parametrically $$\frac{dN_g}{dy\, d^2\mathbf{x}_\perp\, d^2\mathbf{p}_\perp} = \frac{1}{\alpha_s} f\left(\frac{p_T}{Q_s}\right)$$

Note: Q_s depends on $y/\sqrt{s}$

With full nonlinear CYM integrable spectrum

Here: Wilson lines from JIMLWK

T.L. Phys. Lett. B **703** (2011) 325



(Here: y is the amount of evolution: $y = 0$ is MV model initial condition.
At midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Dilute limit and k_T-factorization

Equations of motion solvable in the **dilute limit**; (This is a CGC theorist's "pp collision")
Linearized equations are wave equations (Recall $A^\pm = \pm x^\pm A^\eta$; $A_\eta = -\tau^2 A_\eta$)

$$\left(\tau^2\partial_\tau^2 + \tau\partial_\tau + \tau^2\mathbf{k}_\perp^2\right) A_i(\tau,\mathbf{k}_\perp) = 0$$
$$\left(\tau^2\partial_\tau^2 - \tau\partial_\tau + \tau^2\mathbf{k}_\perp^2\right) A_\eta(\tau,\mathbf{k}_\perp) = 0.$$

$$\Longrightarrow \quad A_i(\tau,\mathbf{k}_\perp) = A_i(\tau=0,\mathbf{k}_\perp)J_0(|\mathbf{k}_\perp|\tau) \qquad A^\eta(\tau,\mathbf{k}_\perp) = -\frac{1}{\tau|\mathbf{k}_\perp|}A^\eta(\tau=0,\mathbf{k}_\perp)J_1(|\mathbf{k}_\perp|\tau).$$

- These are (boost invariant) plane waves $\Longrightarrow$ interpret as particles, gluons.
- Initial fields related to Wilson lines, and thus the unintegrated gluon distribution $\varphi(k_T)$

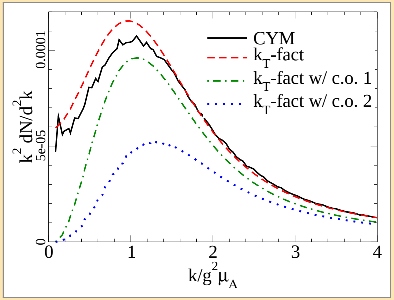
Number spectrum **in the dilute limit**: k_T-factorization formula. (Note: now not integrable)

$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\mathbf{k}_\perp} = \frac{\alpha_s}{S_\perp}\frac{2}{C_F}\frac{1}{{k_T}^2}\int \mathrm{d}^2\mathbf{q}_\perp \varphi^{\mathrm{dip}}(\mathbf{q}_\perp)\varphi^{\mathrm{dip}}(|\mathbf{k}_\perp - \mathbf{q}_\perp|).$$

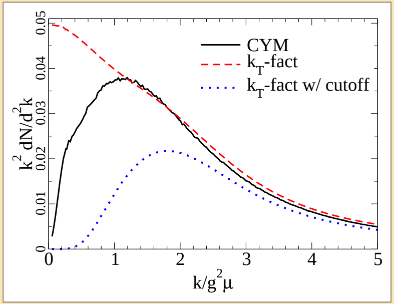
This calculation can also be repeated by assuming that **one** of the two colliding objects is dilute (Theorist's "pA") — **It does not work in "AA"**

CYM vs. $\mathbf{k}_\perp$-factorization

- In fact, also in "AA" the k_T-factorization formula works for high p_T
- Sometimes people also use k_T-factorization with different cutoffs



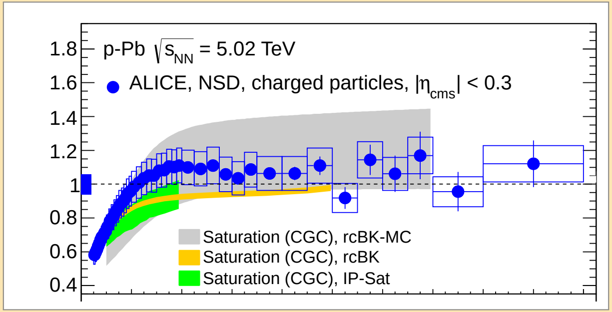
pA: $\mathbf{k}_\perp$-factorization works



AA: k_T-factorization only for large p_T

(Here one proposed cutoff scheme $\frac{dN}{d^2\mathbf{p}_\perp\, dy} = \frac{1}{\alpha_s}\frac{1}{\mathbf{p}_\perp^2}\int_{\mathbf{k}_\perp}\left[\theta(p_T - k_T)\right]\phi_y(\mathbf{k}_\perp)\phi_y(\mathbf{p}_\perp - \mathbf{k}_\perp)$)

Back to R_{pA}



Theory predictions here: fragmentation function for $g \to$hadrons + k_T-factorization formula:

$$\frac{dN}{dy\, d^2\mathbf{k}_\perp} = \frac{\alpha_s}{S_\perp}\frac{2}{C_F}\frac{1}{k_T{}^2}\int d^2\mathbf{q}_\perp \varphi^{\mathrm{dip}}(\mathbf{q}_\perp)\varphi^{\mathrm{dip}}(|\mathbf{k}_\perp - \mathbf{q}_\perp|),$$

- Can also rederive hybrid formula from this, in limit $Q_{s,A} \gg Q_{s,p}$, i.e. $|\mathbf{k}_\perp - \mathbf{q}_\perp| \gg |\mathbf{q}_\perp|$

Conclusions

- Recall conceptual chain here: DIS $\Longrightarrow$ Wilson line $\Longrightarrow$ Glasma fields: energy-momentum tensor and gluon spectrum at initial stages of heavy ion collision
- What then? Configuration very anisotropic: need to go beyond strict classical field limit
  - Plasma instabilities
  - Thermalization in kinetic theory "bottom-up"
- Practical applications?
  - Need transverse geometry from exclusive DIS (see Nestor's lectures)
  - IPglasma takes "IPsat" parametrization of dipole cross section + MV model
    $\Longrightarrow$ CYM numerics, matching energy density to hydrodynamics
- Further topics:
  - Multigluon correlations, initial state ridge correlation (pp, pA) . . .
  - Overoccupied fields $\Longrightarrow$ Chern-Simons charge, chiral magnetic effects . . .