JET SUPPRESSION AND ENERGY LOSS

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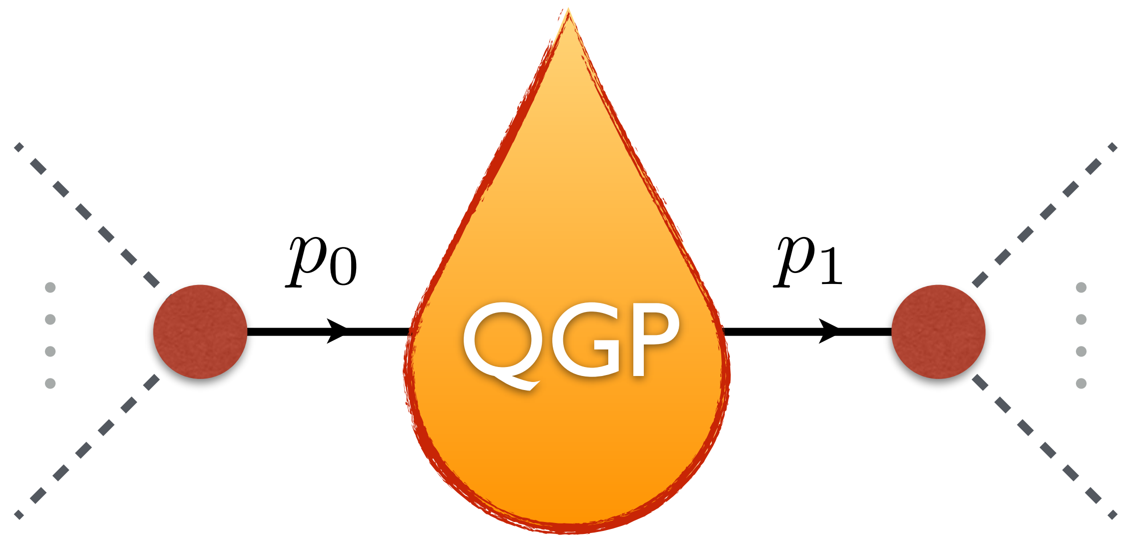
2) INTERACTIONS WITH THE MEDIUM

- dressed propagators
- medium modeling
- broadening
- medium-induced spectrum

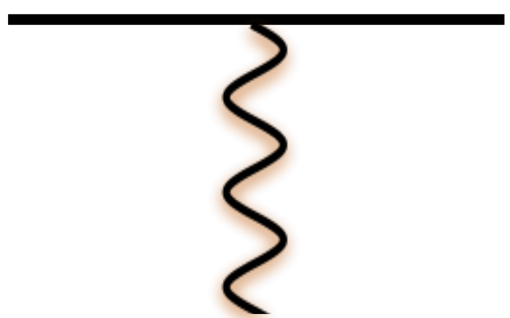
Gluon propagator



$$\mathcal{M}(\{p_{\rm out}\},\{p_{\rm in}\}) = \int \frac{{\rm d}^4 p_1}{(2\pi)^4}\frac{{\rm d}^4 p_0}{(2\pi)^4}(2\pi)^4\delta(p_{\rm in}-p_0)(2\pi)^4\delta(p_1-p_{\rm out})$$
$$\times\, V_\mu(\{p_{\rm out}\},p_1)\, G^{\mu\nu}(p_1,p_0)\, V_\nu(p_0,\{p_{\rm in}\})$$

Interactions with hot & dense QGP: modification of momenta, color flow, tensorial structure, ...?

INTERACTION VERTEX



$$= d^{j\nu}(k) A^{\mu,a}(p-k) V^{abc}_{\mu\nu\rho}(k-p,-k,p) d^{\rho l}(p)$$
$$= g f^{abc}\, 2p^+ A^{-,a}(p-k)\, \delta^{jl}$$

Unified interaction vertex for quark/gluon:

$$\Gamma_{\rm med} = ig\, 2p^+\, \mathbf{T}\cdot\mathcal{A}(p-k)$$

$$\mathbf{T}^a \equiv \mathbf{t}^a_{ij}$$

$$\mathbf{T}^a \equiv i f^{bac}$$

$$A^\mu = \delta^{\mu -}\mathcal{A}$$

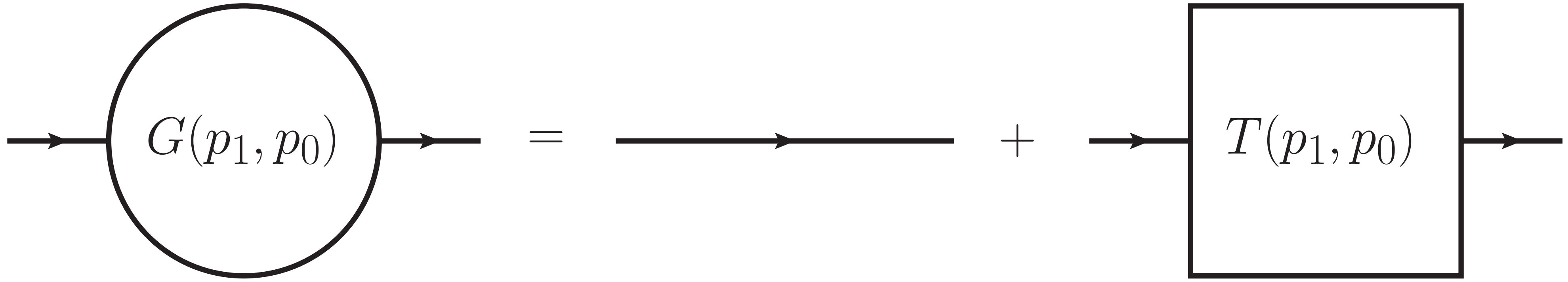
$$\mathcal{A}(q) = 2\pi\delta(q^+) \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{-q^- t} \mathcal{A}(t, \boldsymbol{q})$$

No elastic energy loss in the medium; effect of Lorentz contraction of the background field, boosted in the opposite direction to the projectile.

DRESSED PROPAGATOR

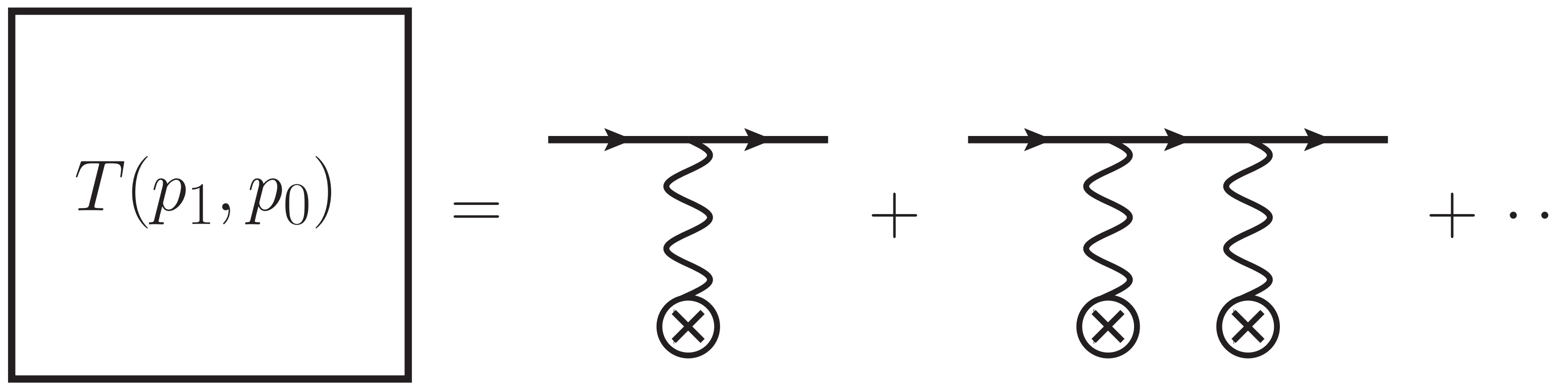
$$G^{\mu\nu}(p_1, p_0) = (2\pi)^4 \delta(p_0 - p_1) d^{\mu\nu}(p_0) G_0(p_0) - d^{\mu i}(p_1) G(p_1, p_0) d^{i\nu}(p_0)$$

- longitudinal modes remain "static"
- transverse modes get dressed!
- Dyson-Schwinger equation for the scalar propagator
  - introduce the transfer matrix



$$G(p_1, p_0) = (2\pi)^2 \delta(p_0 - p_1) G_0(p_0) + G_0(p_1) T(p_1, p_0) G_0(p_0)$$

T-MATRIX EXPANSION



$$T^{(1)}(p_1, p_0) = 2\pi\delta(E_0 - E_1)\, 2E_0 \int_{\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{-i(p_0^- - p_1^-)t}\, ig\mathcal{A}(t, \boldsymbol{p}_0 - \boldsymbol{p}_1)$$

$$T^{(2)}(p_1, p_0) \sim \int_{\infty}^{\infty} \mathrm{d}t_1 \int_{t_1}^{\infty} \mathrm{d}t_2\, \mathrm{e}^{-ip_0^- t_1 + ip_1^- t_2}\, ig\mathcal{A}(t_2, \boldsymbol{q} - \boldsymbol{p}_1)\, \mathrm{e}^{-i\frac{\boldsymbol{q}^2}{2E_0}(t_2 - t_1)}\, ig\mathcal{A}(t_1, \boldsymbol{p}_0 - \boldsymbol{q})$$

Relatively simple recurrent structure appears….

THE RETARDED PROPAGATOR

$$\int \frac{\mathrm{d}p_1^-}{2\pi} \int \frac{\mathrm{d}p_0^-}{2\pi}\, \mathrm{e}^{-ip_1^- t_1 + ip_0^- t_0} G(p_1, p_0) = 2\pi\delta(E_0 - E_1)\, \frac{-1}{2E_0} \mathcal{G}(\boldsymbol{p}_1, t_1; \boldsymbol{p}_0, t_0)$$

after the dust has settled, this is what we are interested in!

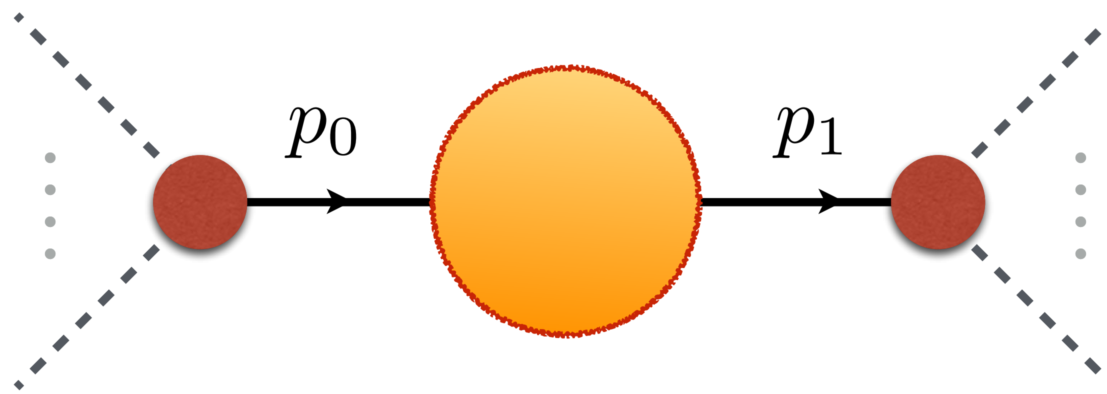
$$\mathcal{G}(\boldsymbol{p}_1, t_1; \boldsymbol{p}_0, t_0) = (2\pi)^\delta (\boldsymbol{p}_0 - \boldsymbol{p}_1) \mathrm{e}^{-i\frac{\boldsymbol{p}_0^2}{2E}(t_1 - t_0)}$$

$$+ ig \int_{t_0}^{t_1} \mathrm{d}t \int_{\boldsymbol{q}} \mathrm{e}^{-i\frac{\boldsymbol{p}_1^2}{2E}(t_1 - t)} \mathcal{A}(t, \boldsymbol{q}) \mathcal{G}(\boldsymbol{p}_1 + \boldsymbol{q}, t; \boldsymbol{p}_0, t_0)$$

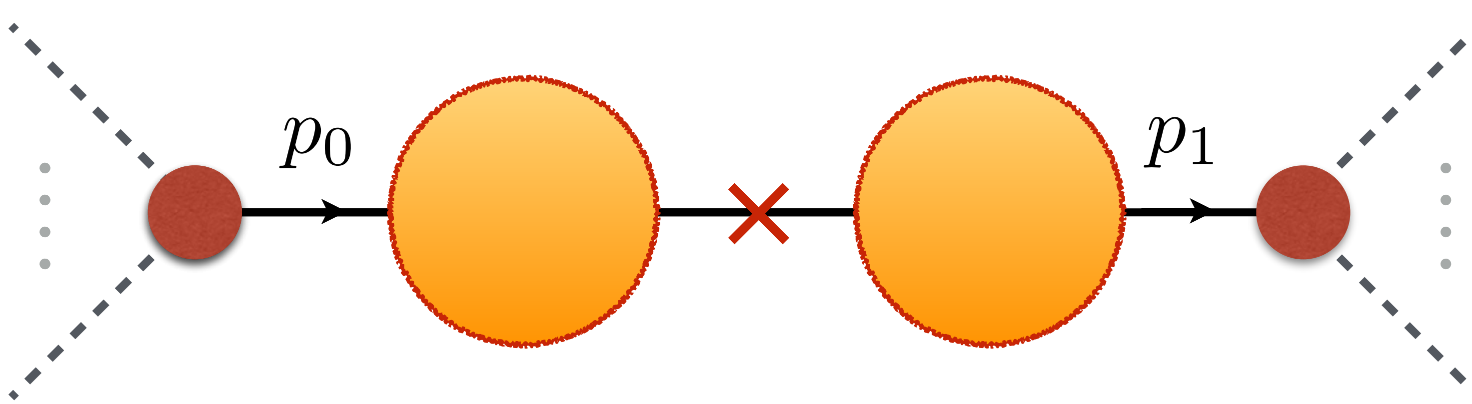
Simple recurrent structure: propagation+interaction+…

Same general structure goes through for more complicated interactions with the medium.

INTERNAL VS EXTERNAL PROPAGATOR

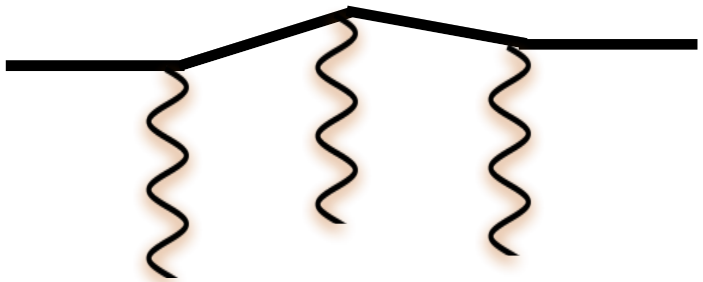


$$\mathcal{M}(\{p_{\rm out}\},\{p_{\rm in}\}) = (2\pi)\delta(E_0 - E_1)\int_{-\infty}^{\infty} \mathrm{d}t_1 \int_{-\infty}^{\infty} \mathrm{d}t_0$$
$$\times\, \mathrm{e}^{ip^-_{\rm out}t_1}\Gamma^i(\{p_{\rm out}\},p)\,\frac{1}{2E_0}\mathcal{G}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0)\,\mathrm{e}^{-ip^-_{\rm in}t_0}\Gamma^i(\boldsymbol{p}_0,\{p_{\rm in}\})$$



$$\mathcal{M}(p_1,\{p_{\rm in}\}) = (2\pi)\delta(E_0 - E_1)\mathrm{e}^{i\frac{\boldsymbol{p}_1^2}{2E}L}\int_{-\infty}^{\infty} \mathrm{d}t_0\; \mathcal{G}(\boldsymbol{p}_1, L;\boldsymbol{p}_0,t_0)\,\mathrm{e}^{-ip^-_{\rm in}t_0}\Gamma^i(\boldsymbol{p}_0,\{p_{\rm in}\})$$

In coordinate space…



$$\mathcal{G}(\vec{x}_1, \vec{x}_0) = \mathcal{G}_0(\vec{x}_1 - \vec{x}_0) + \int_{\vec{x}} \mathcal{G}_0(\vec{x}_1 - \vec{x})\, ig\mathcal{A}(\vec{x})\, \mathcal{G}(\vec{x}, \vec{x}_0)$$

Another way of seeing it: $\mathcal{G}$ is the Greens function.

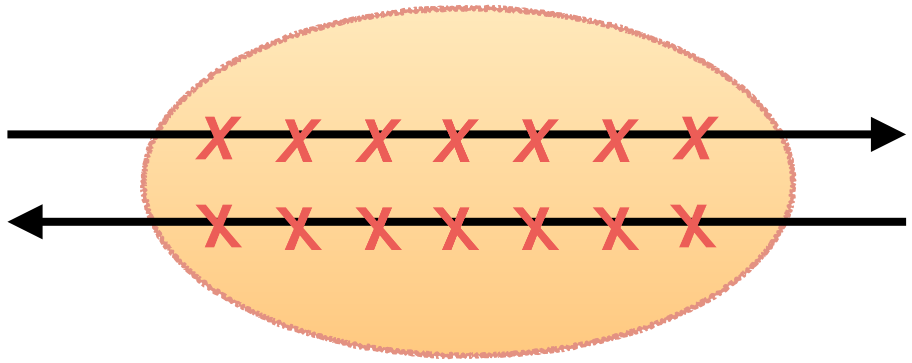
$$\mathcal{G}(\vec{x}_1, \vec{x}_0) = \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t_1)=\boldsymbol{x}_1} \mathcal{D}\boldsymbol{r}\, \mathrm{e}^{i\frac{E}{2}\int_{t_0}^{t_1} \mathrm{d}s\, \dot{\boldsymbol{r}}^2(s)} U(t_1, t_0; [\boldsymbol{r}(s)])$$

$$U(t_1, t_0; [\boldsymbol{r}]) = \mathcal{P}\exp\left[ig\int_{t_0}^{t_1} \mathrm{d}s\, \mathcal{A}(s, \boldsymbol{r}(s))\right]$$

- 1st term kinetic energy
- path ordered exponential (Wilson line)

MEDIUM-INDUCED BROADENING

Lowest order (non-trivial) process: single parton!



$$\mathcal{M}^a(p_1,p_0) = \mathcal{G}(\boldsymbol{p}_1, L; \boldsymbol{p}_0, 0)$$

$$\langle \mathcal{M}^a(p_1,p_0)\mathcal{M}^{*,a}(\bar{p}_0,p_1)\rangle = \frac{1}{N_c^2-1}\langle \mathrm{Tr}\,\mathcal{G}(\boldsymbol{p}_1, L; \boldsymbol{p}_0, 0)\mathcal{G}^\dagger(\bar{\boldsymbol{p}}_0, 0; \boldsymbol{p}_1, L)\rangle$$
$$\propto (2\pi)^2\delta(\boldsymbol{p}_0 - \bar{\boldsymbol{p}}_0)$$

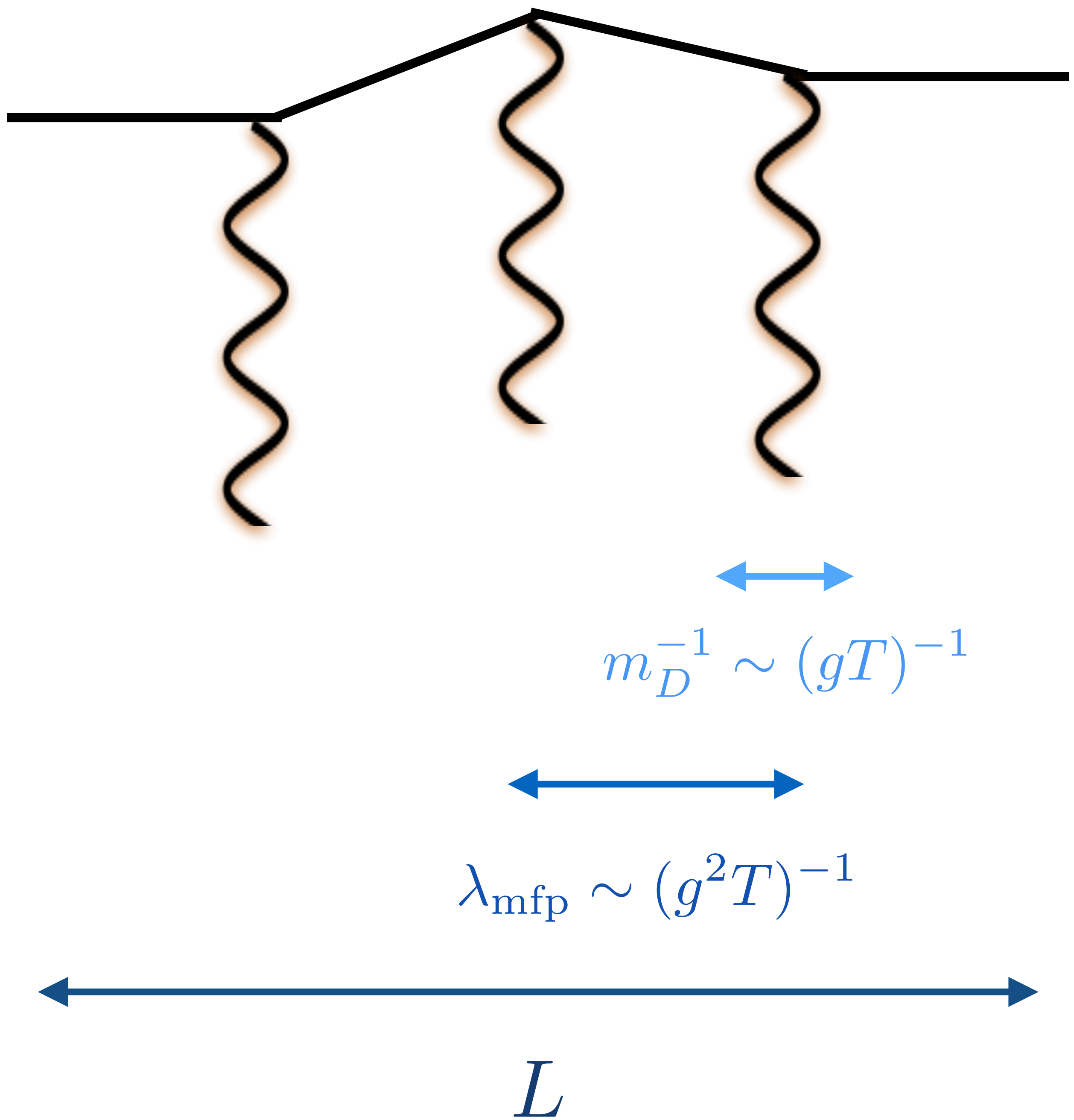
Reflects translational invariance.

After Fourier transform:

$$= \int \mathcal{D}\boldsymbol{r}_1(\mathcal{D}\boldsymbol{r}_2)^* \exp\left\{ i\frac{E}{2}\int_{t_0}^{t_1} \mathrm{d}s\,(\dot{\boldsymbol{r}}_1^2(s) - \dot{\boldsymbol{r}}_2^2(s)) - \frac{N_c n}{2}\int_{t_0}^{t_1}\mathrm{d}s\,\sigma(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right\}$$

target cross section

SCALES OF THE MEDIUM



- weak-coupling
- medium consists of static scattering centers
- thermal distribution $n \sim T^3$
- separation of scales!

$$\sigma_{\rm el} \sim g^4/m_D^2$$

$$\lambda_{\rm mfp} \sim \frac{1}{n\sigma_{\rm el}}$$

Typically assume a static medium *n=const* (but can generalize)…

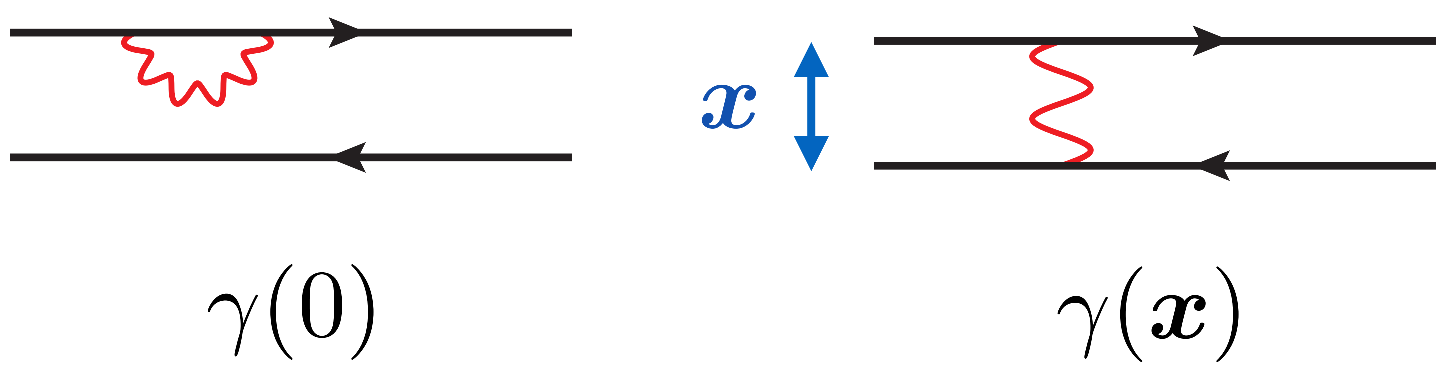
Independent scattering

Have to average over medium configurations.

$$\langle \mathcal{A}^a(t_1, \boldsymbol{x}_1)\mathcal{A}^{*,b}(t_2, \boldsymbol{x}_2)\rangle = n\delta^{ab}\,\delta(t_1 - t_0)\gamma(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

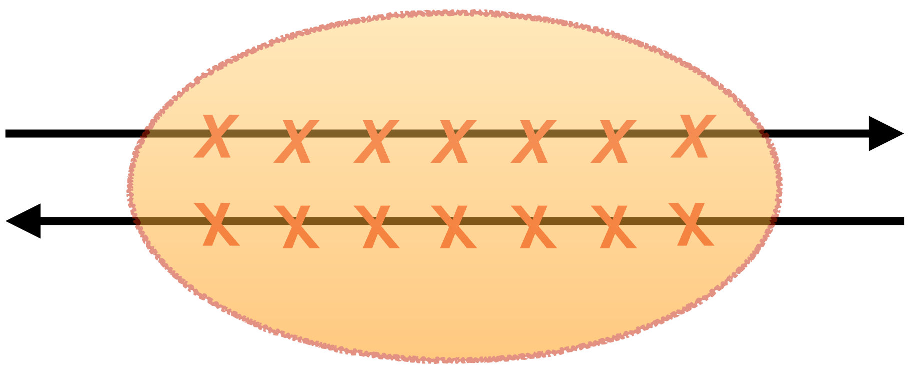
$$\gamma(\boldsymbol{x}) = g^2 \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\,\frac{\mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$

Sensitive to the transverse extension of the "dipole".



$$\sigma(\boldsymbol{x}) = 2g^2\left[\gamma(0) - \gamma(\boldsymbol{x})\right]$$

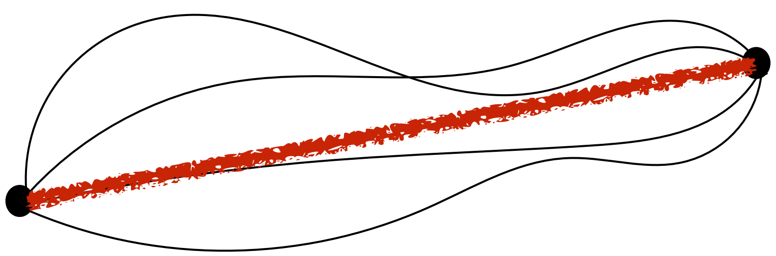
FINAL SPECTRUM



$$= (\boldsymbol{x}_1|\mathcal{G}_0(t_1,t_0)|\boldsymbol{x}_0)(\bar{\boldsymbol{x}}_0|\mathcal{G}_0^\dagger(t_0,t_1)|\bar{\boldsymbol{x}}_1)$$

$$\times \exp\left[-\frac{N_c n}{2}\int_{t_0}^{t_1} \mathrm{d}s\,\sigma(\boldsymbol{v}_{\rm cl}(s))\right]$$

Evaluation of path integral singles out the classical trajectory!



$$\boldsymbol{v}_{\rm cl}(s) = \frac{s-t_0}{t_1-t_0}(\boldsymbol{x}_1-\bar{\boldsymbol{x}}_1) + \frac{t_1-s}{t_1-t_0}(\boldsymbol{x}_0-\bar{\boldsymbol{x}}_0)$$

Final spectrum a convolution of hard process & medium modification (separation of scales)

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega_p} = \int \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\,\mathcal{P}(\boldsymbol{p},L;\boldsymbol{k},0)\,\frac{\mathrm{d}N}{\mathrm{d}\Omega_k}$$

BROADENING PROBABILITY

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{p},t) = -\frac{N_c n}{2}\int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\,\sigma(\boldsymbol{q})\mathcal{P}(\boldsymbol{p}-\boldsymbol{q},t)$$

The path integral for "dipole" can be recast as an evolution equation of the broadening probability [dynamic medium *n=n(t)*].

Solution

$$\mathcal{P}(\boldsymbol{p},t) = \int \mathrm{d}^2\boldsymbol{x}\,\exp\left[-\frac{N_c n}{2}\sigma(\boldsymbol{x})t - i\boldsymbol{p}\cdot\boldsymbol{x}\right]$$

For $q \ll p$ we perform a gradient expansion, to obtain a Fokker-Planck equation with diffusion coefficient $\hat{q}$

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{p},t) \simeq -\frac{1}{4}\left(\frac{\partial}{\partial \boldsymbol{p}}\right)^2\left[\hat{q}(\boldsymbol{p}^2)\mathcal{P}(\boldsymbol{p},t)\right]$$

$$\hat{q}(\boldsymbol{p}^2) = -\frac{N_c n}{2}\int^{\boldsymbol{p}^2}\frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\,\boldsymbol{q}^2\sigma(\boldsymbol{q})$$
$$\simeq \frac{g^4 N_c\, n}{4\pi}\ln\frac{\boldsymbol{p}^2}{m_D^2}$$

TRANSVERSE MOMENTUM DIFFUSION

Neglect logarithmic momentum dependence $\hat{q}=const.$

$$\mathcal{P}(\boldsymbol{p}, L) = \frac{4\pi}{\hat{q}L} \mathrm{e}^{-\frac{\boldsymbol{p}^2}{\hat{q}L}}$$

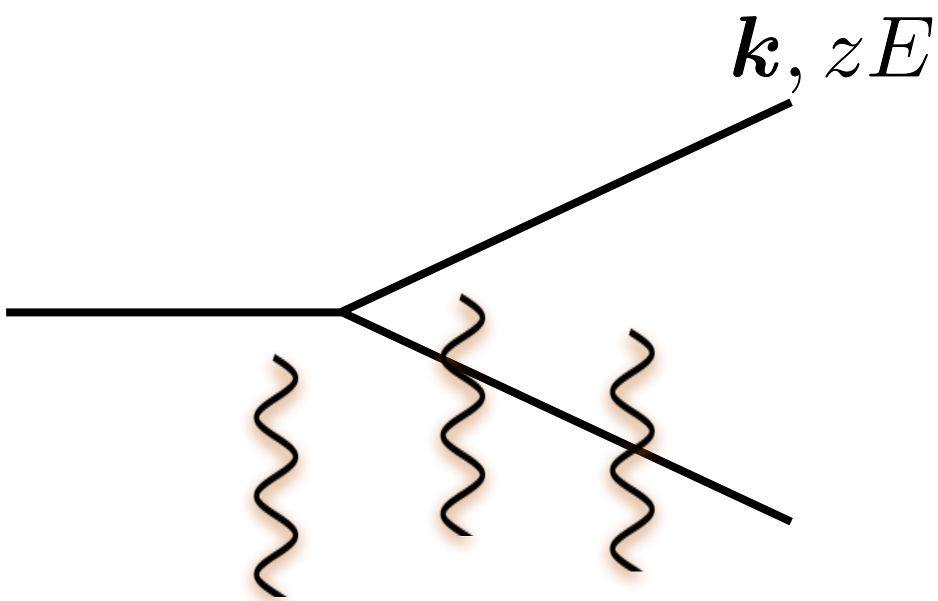
$$\langle p_\perp^2 \rangle = \hat{q}L$$

Gaussian momentum broadening.

In the weak-coupling picture of the medium, the parameter $\hat{q}$ governs all physical processes (broadening, radiative processes, decoherence). Can be accessed by comparing to experimental data!

IN-MEDIUM RADIATION

Particles undergo diffusion in the medium - but how do y lose energy? Radiative processes!



$$\left.\begin{aligned} \langle \boldsymbol{k}^2 \rangle &= \hat{q} t_{\rm f} \\ t_{\rm f} &\sim \frac{\omega}{\langle \boldsymbol{k}^2 \rangle} \end{aligned}\right\} \quad \begin{aligned} k_{\rm br}^2 &= \sqrt{\hat{q}\omega} \\ t_{\rm br} &= \sqrt{\frac{\omega}{\hat{q}}} \end{aligned}$$

Landau-Pomeranchuk-Migdal effect

BDMPS-Z spectrum

$$\omega \frac{{\rm d}N}{{\rm d}\omega} = \frac{\alpha_s N_c}{\pi} \frac{L}{t_{\rm f}} = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q} L^2}{\omega}}$$

Maximal energy $\omega_c = \hat{q} L^2$ corresponds to $t_{br} = L$ - maximal effect of scattering with the medium.

Two regimes

Characteristic angle of radiation $\theta_{\rm br}(\omega) \sim k_{\rm br}/\omega \sim (\hat{q}/\omega^3)^{1/4}$

Multiplicity of gluons $N(\omega) = \int_\omega^\infty {\rm d}\omega' \, \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\sqrt{\frac{\hat{q}L^2}{\omega}}$

Energy loss: $\Delta E = \int_0^\infty {\rm d}\omega \, \omega \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\hat{q}L^2$

$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$

can become significant for large medium!

rare, small-angle emission $\omega_c = \hat{q}L^2$

$$\theta_{\rm br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

copious, large-angle emissions $\omega_s = \bar{\alpha}^2\hat{q}L^2$

$$\theta_{\rm br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

SUMMARY

- at high energies, the medium interactions can be modeled by quasi-instantaneous exchanges
- radiative processes can lead to energy loss
- tomorrow we will discuss the implications on experimental data