Theory of the Deconfinement Transition and its Signatures
- Lecture 1 -

Chihiro Sasaki
Institute of Theoretical Physics
University of Wroclaw

Outline

Lecture 1: Brief introduction

Lecture 2: Interacting quarks and gluons

- ❑Polyakov-loop model: deconfinement
- ❑Nambu--Jona-Lasinio model: chiral SB

Lecture 3: Critical behaviors

- ❑Phase transition and the Landau theory
- ❑Fluctuations of conserved charges

Main scope

- **Strategy**: ideal situations as much as possible!
  - Massless limit
  - Infinitely heavy-mass limit
  - Thermodynamic limit
  - Uniform systems
- **Aim**
  - Solvable, (semi)-analytic and intuitive
  - The essence must be kept - symmetries and naturalness as our guide

1. Introduction

QCD: The theory of strong int.

- Quarks
  - Spin ½ fermions
  - 6 flavors $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$ ← Q = 2/3, ← Q = -1/3

$$\underbrace{\underset{4\,\mathrm{MeV}}{m_u} \lesssim \underset{7\,\mathrm{MeV}}{m_d} < \underset{100\,\mathrm{MeV}}{m_s}}_{\text{light quarks}} \ll \underbrace{\underset{1.2\,\mathrm{GeV}}{m_c} \ll \underset{4\,\mathrm{GeV}}{m_b} \ll \underset{180\,\mathrm{GeV}}{m_t}}_{\text{heavy quarks}}$$

- Gluons
  - Spin 1 bosons
  - Massless (similar to photons in QED)

QCD: The theory of strong int.

- Lagrangian

$$\mathcal{L} = \bar{q}_f^a \left(i\gamma_\mu D^\mu_{ab} - \delta_{ab} m_f \right) q_f^b - \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha$$

colors $= (r\,, g\,, b)$ and flavors $= (u, d, s, c, b, t)$

- Quarks carry 3 types of color quantum number – Red, Green, Blue
- Gluons responsible for color exchange:

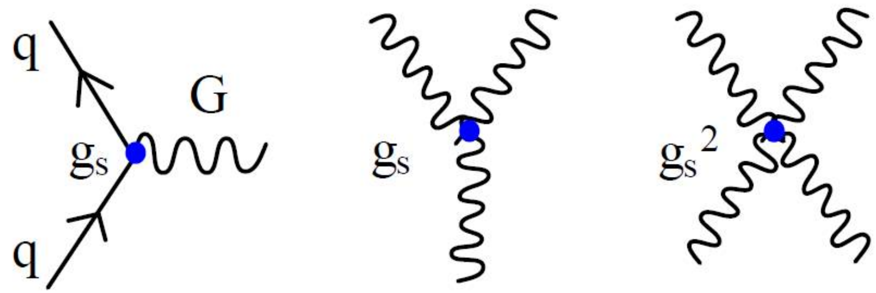
  3 (colors) x 3 (anti-colors) – 1 (white) = 8

QCD as SU(3) gauge theory

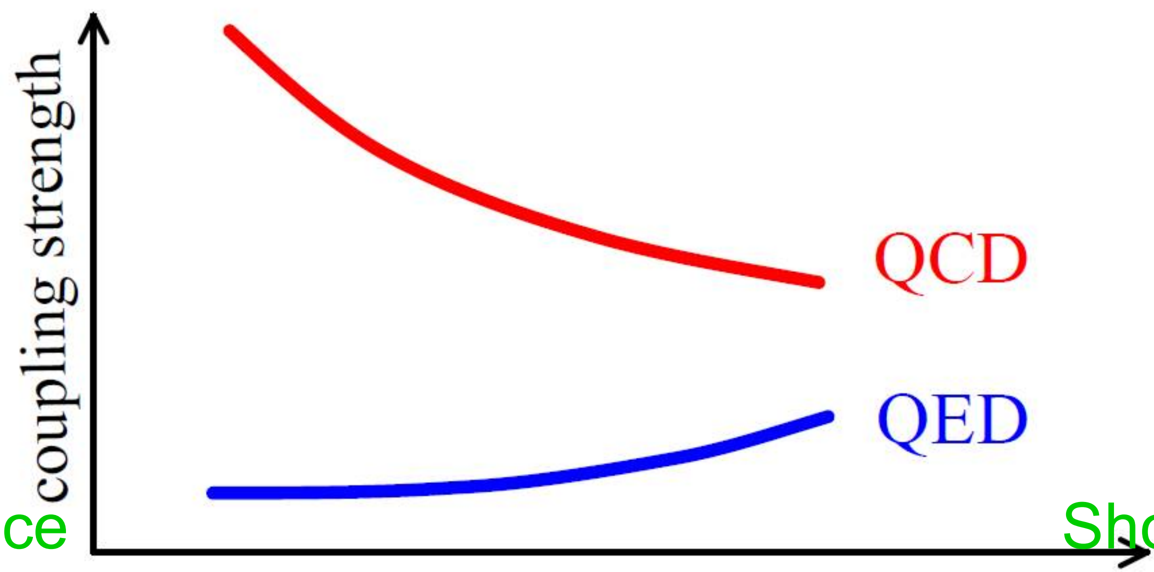
	QED	QCD
Matter fermions	Electrons	Quarks
Gauge bosons	Photons	Gluons
Gauge group	U(1)	SU(3)
Coupling strength	$e^2/4\pi$	$g^2/4\pi$

Asymptotic freedom

- In contrast to photons, gluons self-interact.



- Asymptotic freedom



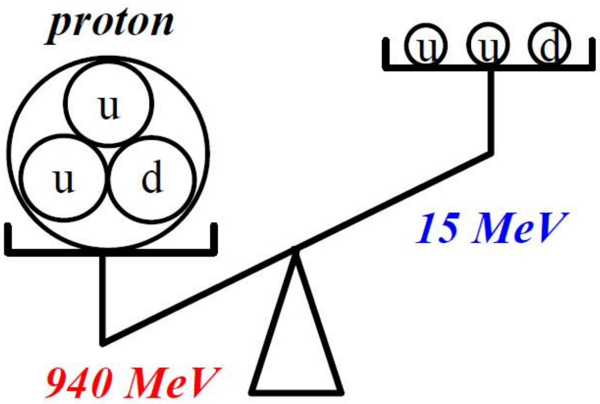
Consequences of large α s

❑Confinement

- Hadrons as colorless states ≈ 300 >> 5
  - ➢Baryons: fermions, (mostly) qqq bound states … 160
  - ➢Mesons: bosons, (mostly) qqbar bound states … 130

❑Dynamical breaking of chiral symmetry

- Heavy-quark bound states ≈ hydrogen atoms bound by the Coulomb force due to 1-gluon exch.
- Light-quark bound states
- Strong int. generates qqbar condensation.



Scales and symmetries

❑Scales in strong interactions

- Masses: mpi = 140 MeV, mN = 940 MeV
- Sizes: rpi = 0.7 fm, rN = 0.9 fm (1 fm = 10^-15 m)
- Cf. Bohr radius = atom size = 5x10^4 fm

❑Selected global symmetries

- Pure YM: Z(3) symmetry (center of SU(3))
- Light-quark sector: chiral symmetry (no L-R)
- Heavy-quark sector: spin-flavor symmetry
  - ➢Heavy-light systems: e.g. D, Ds, B, Bs mesons constrained by both symmetries.

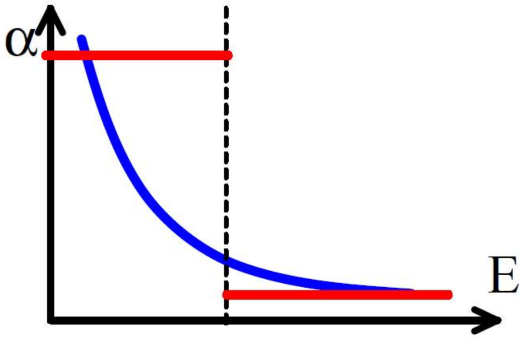
2. Hot QCD

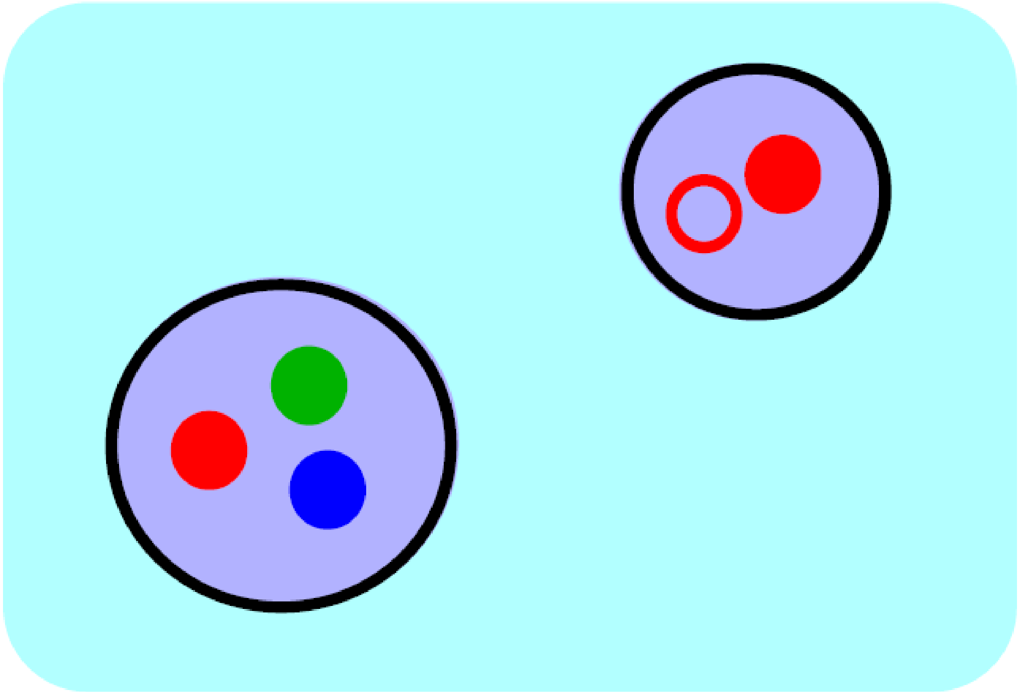
When the system is heated…

- At T=0; the ground state is characterized by
  - Confinement
  - Chiral symmetry breaking
- At a high T, it changes its character:
  - Deconfinement
  - Chiral symmetry unbroken
- QCD phase transition and the phase structure
  - Nature of new state(s) of matter?
  - From **Hadrons** to the **Quark**-**Gluon** **Plasma**?

The bag model [Chodos et al. (1974)]

- ❑ Hadrons as bags in the non-pert. vacuum
- ❑ Quarks and gluons are treated perturbatively.
- ✓ The physical vacuum must have the lowest energy – the ground state
- ✓ E(non-pert.) ≡ 0 and E(pert) = B>0
- ✓ B: bag constant





The bag model [Chodos et al. (1974)]

- ❑The hadron has the energy (R: radius the bag)

$$E_h = VB + k/R\,, \quad V = (4\pi/3)R^3 \quad (*)$$

→The hadron radius by minimizing the energy;

$$\frac{dE_h}{dR} = 4\pi R^2 B - k/R^2 = 0 \text{ thus } R_h = (k/4\pi B)^{1/4}$$

→The hadron mass from eq.(*);

$$M_h = \frac{16\pi}{3}(k/4\pi)^{3/4}B^{1/4}$$

- ❑B.C.(no leak of color charge on the bag surface) & Dirac eq. → k ≈ 2 x nq (nq=3 for N)

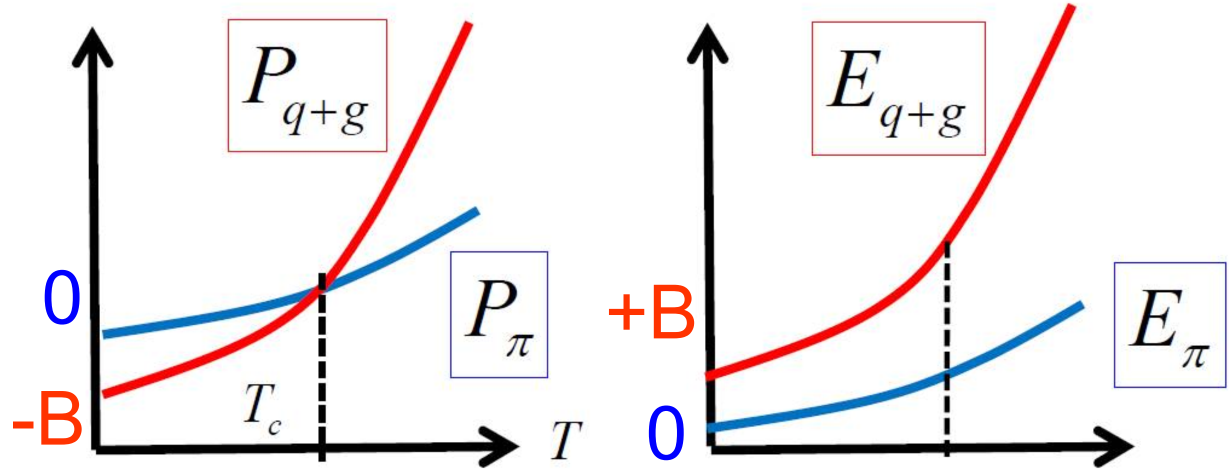
→ B^¼ = 0.11 GeV, Rh = 1.47 fm [cf. Rn = 0.9 fm]

More realistic B^¼ ≈ 0.2 GeV Not too bad!

Bag equation of state

- ❑A toy model: massless pions (conf.) & massless quarks and gluons (deconf.)
- ❑Partition functions

$$T \ln Z_{B/F} = \mp d_{B/F} V T \int \frac{d^3k}{(2\pi)^3} \ln(1 \mp e^{-E/T})$$



$$P_\pi(T_c) = P_{q+g}(T_c)$$

$$T_c^4 = \frac{90}{\pi^2} \frac{B}{d_{\rm QGP} - d_\pi} \simeq (144\,{\rm MeV})^4$$

Tc okay, but a 1st order

Beyond the bag model I

- $P(T) = \int_0^T dt\, s(t)\,, \quad \epsilon(T) = Ts(T) - P(T)$

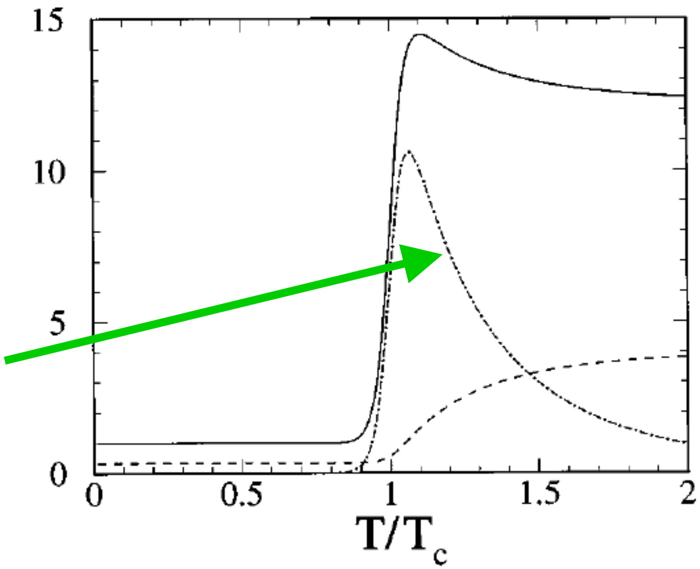
→ When s(T) is given, all others are obtained.

- A parameterization [Asakawa and Hatsuda (1997)]

$$\begin{aligned} s(T) &= f(T)s_{\pi}(T) + (1 - f(T))s_{\rm QGP}(T) \\ f(T) &= (1/2)\left[1 - \tanh((T - T_c)/\Gamma)\right] \end{aligned}$$

→A smooth crossover

with Γ /Tc = 0.05



Beyond the bag model II

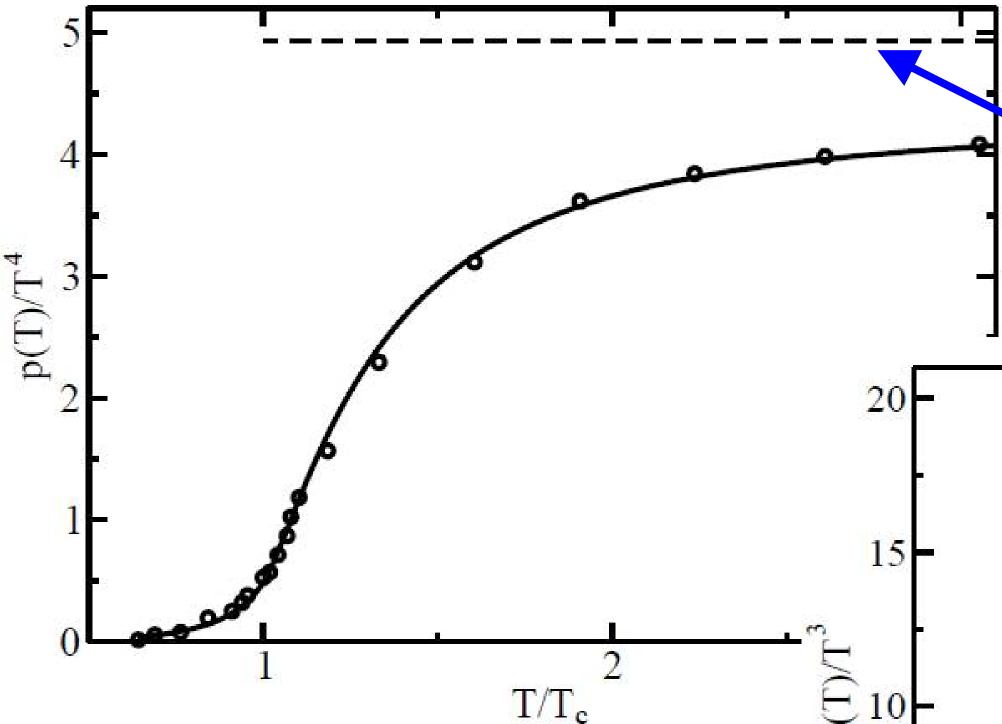
- ❑ Quasi-particle excitations [DeTar ('85); Goloviznin and Satz ('93); Peshier et al. ('94); Gorenstein and Yang ('95)]

→ They obey the dispersion relation;

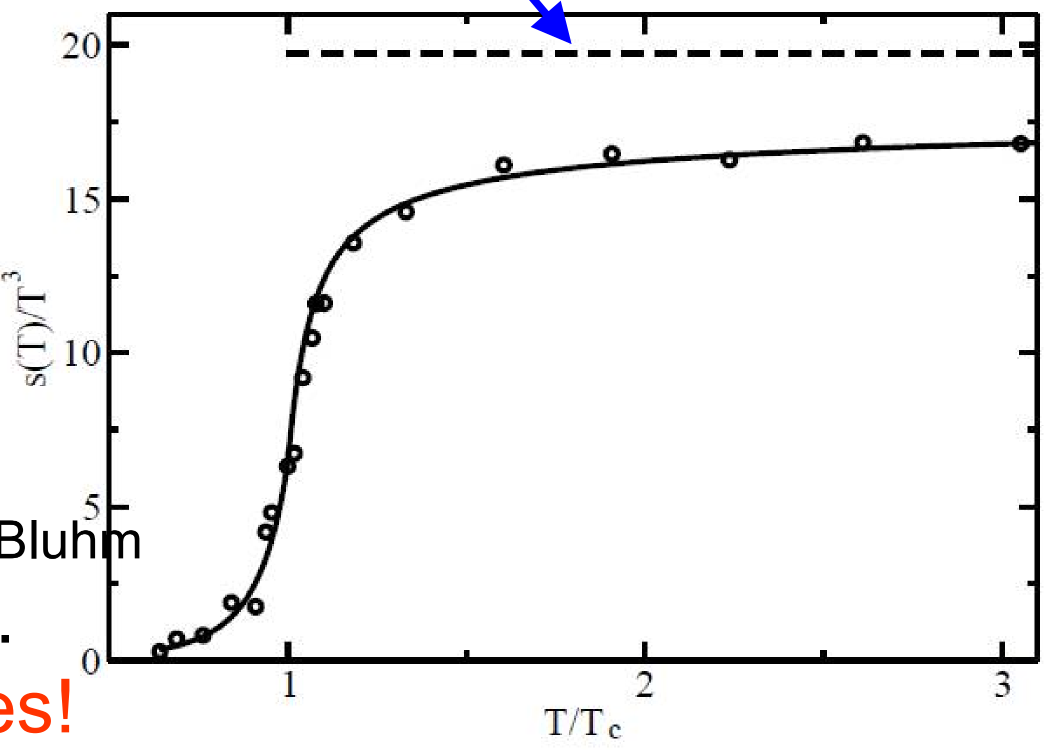
$$\omega^2 = \vec{p}^2 + m^2 + \Pi(T,\mu) = \vec{p}^2 + M^2(T,\mu)$$

- ❑ Thermodynamics: extrapolation from pQCD
  - Eff.masses: Mq = Cq gT, Mg = Cg gT
  - Thermodynamic consistency determines the bag function B(T).
  - Running QCD coupling g(T): 2-loop β w/ phenomenological assumptions.

Quasi-particle EoS for Nf=2+1



Stefan-Boltzmann limit



- Model vs. Lattice QCD

  Figures from PhD thesis of M.Bluhm

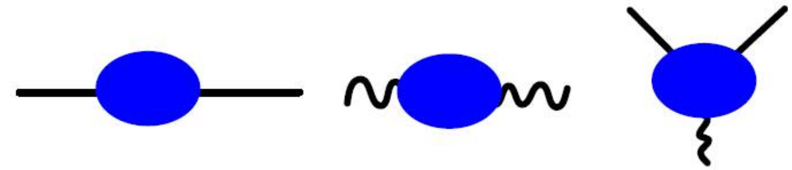
- Not self-consistent calc.

→ NP effects, symmetries!

$$\kappa \frac{\partial g}{\partial \kappa} = \beta(g) = -\beta_0 g^3 - \beta_1 g^5 + \cdots$$

$$\Rightarrow \quad \alpha_s(\kappa) = \frac{g^2(\kappa)}{4\pi}$$

QPM I



- Running coupling

$$G^2_{2-\mathrm{loop}}(T) = \frac{16\pi^2}{\beta_0 \log \zeta^2}\left(1 - \frac{2\beta_1}{\beta_0}\frac{\log(\log \zeta^2)}{\log \zeta^2}\right)$$

$$\zeta = \lambda(T - T_s)/T_c, \; \beta_0 = 11 - \frac{2}{3}N_f \text{ and } \beta_1 = 51 - \frac{19}{3}N_f$$

Scale parameter

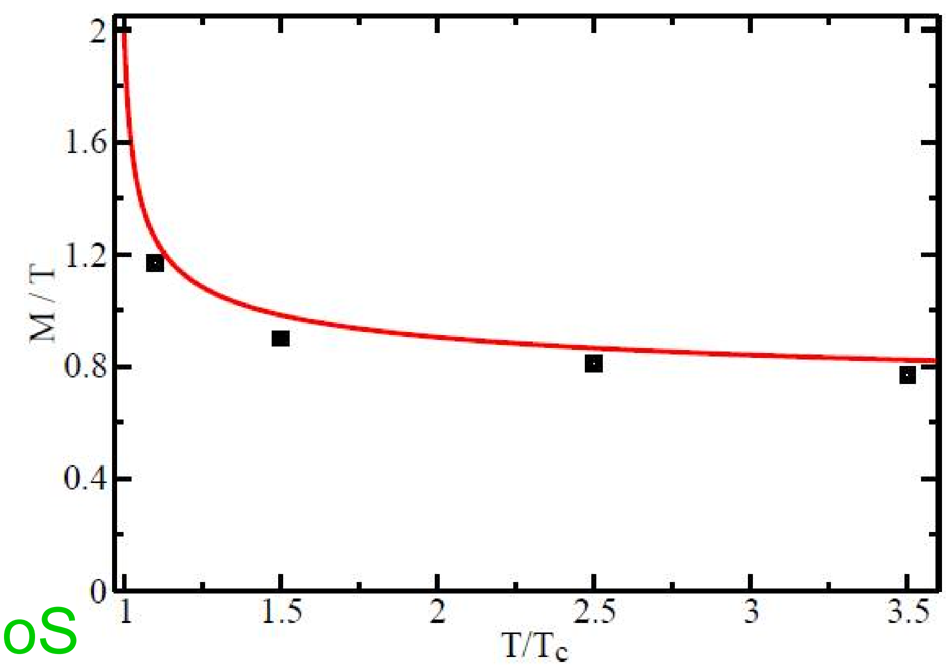
Temperature shift

- Parameters

$$\lambda = 7.8, \; T_s = 0.8\,T_c,$$

$$T_c = 170 \text{ MeV}$$

From Lattice EoS



QPM II

- Pressure

$$p(T,\mu) = \sum_{i=u,d,s,g} p_i(T,\mu_i,M_i(T,\mu)) - B(M_{u,d,s,g}(T,\mu)),$$

↑ Non-interacting forms

- Stationary conditions

$$\left.\frac{\delta p}{\delta M_i^2}\right|_{T,\mu,M^2_{j\neq i}} = 0 \quad \Longleftrightarrow \quad \frac{\partial B}{\partial M_i^2} = \frac{\partial p_i}{\partial M_i^2}$$

→ Entropy density, quark number density

$$n_l = \left.\frac{\partial p}{\partial \mu_l}\right|_T = \frac{d_l}{2\pi^2}\int_0^\infty dk k^2 \left(\frac{1}{[e^{\{\omega_l-\mu_l\}/T}+1]} - \frac{1}{[e^{\{\omega_l+\mu_l\}/T}+1]}\right)$$

Non-interacting form ← T-dep. From M(T) compensated by B(T)

QPM III

- Towards finite μ : Maxwell's relations

$$\frac{\partial^2 p}{\partial \mu \partial T} = \frac{\partial s}{\partial \mu} = \frac{\partial n_q}{\partial T} = \frac{\partial^2 p}{\partial T \partial \mu}$$

$$a_\mu \frac{\partial G^2}{\partial \mu} + a_T \frac{\partial G^2}{\partial T} = b\,.$$

NOTE: a μ , aT, b depend on T, μ , G(T, μ)

- Solving the "flow" equations under the initial condition: G(T, μ =0)